Senior 4 Pre-Calculus Mathematics (40S) Standards Test

Marking Guide

June 2005

Manitoba

Building for the Future

Copyright © 2005, the Crown in Right of Manitoba,
as represented by the Minister of Education, Citizenship and Youth.
Manitoba Education, Citizenship and Youth,
Instruction, Curriculum and Assessment Branch,
Winnipeg, Manitoba R3E 3J5

Permission is hereby given by the copyright owner to reproduce this document on a non-profit basis for educational purposes provided the source is mentioned.

Table of Contents

General Marking Instructions **1**

Scoring Guidelines **3**

Answer Key for Selected-Response Questions 5

Marking Key including student exemplars 7

Part 1: Open-Response Questions 8

Part 2: Restricted-Response Questions 39

Part 2: Open-Response Questions 54

Appendices **81**

Appendix A: Marking Guidelines 83

Appendix B: Irregularities in Standards Tests 85

General Marking Instructions

Please make no marks in the student test booklets. If the booklets have marks in them, the marks need to be removed by department staff prior to sample marking should the booklet be selected.

Please ensure that:

- the student test booklet number and the number on the *Answer/Scoring Sheet* are identical;
- **only a pencil is used to complete the *Answer/Scoring Sheets*, by both students and markers**;
- the totals of each of the four parts are written at the bottom;
- each student's final result is recorded, by test booklet number, on the corresponding ***Answer/Scoring Sheet***;
- the grand total is written in the appropriate location;
- the *Answer/Scoring Sheet* is complete; and
- a photocopy has been made for school records.

Once marking is completed, please forward the ***Answer/Scoring Sheets*** to Manitoba Education, Citizenship and Youth in the envelope provided (for more information see the *Administration Manual for Supervising Teachers*).

Scoring the Selected-Response Questions

Please follow the instructions on the ***Answer Key for Selected-Response Questions*** on page 5.

Scoring the Restricted-Response and Open-Response Questions

The remainder of the Senior 4 Pre-Calculus Mathematics (40S) Standards Test is composed of restricted-response questions and open-response questions. Restricted-response questions are worth 1 mark each and open-response questions are worth 2 to 5 marks each.

Each question is designed to elicit a well-defined response according to the associated specific learning outcome(s) and relevant mathematical processes. Their purpose is to determine whether a student meets the standards for the course as they relate to the knowledge and skills associated with the question.

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Key* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Standards Tests

During the administration of standards tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "NR" and/or "0" only (e.g., student was present but did not attempt any questions) please document this on the *Irregular Test Paper Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education, Citizenship and Youth at the earliest opportunity to advise and/or seek assistance. If required, we will connect you to an advisor. Also, please be sure to notify us before making any modifications to the answer keys or scoring rubrics. Contact person: Chantal Bérard at 945-7590 or toll-free at 1 800 282-8069, extension 7590.

SCORING GUIDELINES

Answer Key for Selected-Response Questions

Answer Key for Selected-Response Questions

If you are using this sheet by punching out holes corresponding to the correct answers and overlaying this sheet on the *Answer/Scoring Sheet*:

1) check the student's responses for multiple bubbles first (questions with multiple bubbles are to be scored as "0");

2) overlay this page on the *Answer/Scoring Sheet*; and

3) count the number of correct responses, excluding questions with multiple responses, if any.

You may also score the selected-response questions by making a transparency of this page and overlaying it on the *Answer/Scoring Sheet*.

Remember to write the total score for the selected-response questions at the bottom of the *Answer/Scoring Sheet*.

SELECTED RESPONSE QUESTIONS / QUESTIONS À RÉPONSE CHOISIE

Fill in the best answer for each question. / Choisir la meilleure réponse pour chaque question.

11	Ⓐ Ⓑ Ⓒ **Ⓓ**	14	Ⓐ **Ⓑ** Ⓒ Ⓓ	17	Ⓐ **Ⓑ** Ⓒ Ⓓ	20	Ⓐ Ⓑ Ⓒ **Ⓓ**	23	Ⓐ **Ⓑ** Ⓒ Ⓓ
12	Ⓐ Ⓑ Ⓒ **Ⓓ**	15	Ⓐ Ⓑ Ⓒ **Ⓓ**	18	**Ⓐ** Ⓑ Ⓒ Ⓓ	21	Ⓐ Ⓑ Ⓒ **Ⓓ**	24	**Ⓐ** Ⓑ Ⓒ Ⓓ
13	Ⓐ Ⓑ **Ⓒ** Ⓓ	16	**Ⓐ** Ⓑ Ⓒ Ⓓ	19	Ⓐ **Ⓑ** Ⓒ Ⓓ	22	Ⓐ Ⓑ **Ⓒ** Ⓓ	25	Ⓐ Ⓑ **Ⓒ** Ⓓ

Selected-Response Questions			
Question	***Outcome***	***Question***	***Outcome***
11	A2	19	H2
12	A2, A4	20	G3
13	A4	21	E3
14	A6, B7	22	E1
15	A1	23	F1
16	B1, B2	24	F3
17	B1	25	F3
18	D5		

SCORING GUIDELINES

Marking Key
including student exemplars

Part 1—Open-Response Questions

Question No. 1

Outcome: D6

Solve the equation $2^{3x+1} = 5^{x+3}$ algebraically.
Express your final answer correct to 3 decimal places.

Solution

Method 1

$$(3x+1)\log 2 = (x+3)\log 5$$ — 1 mark for applying logarithms

$$3x\log 2 + \log 2 = x\log 5 + 3\log 5$$

$$3x\log 2 - x\log 5 = 3\log 5 - \log 2$$

$$x(3\log 2 - \log 5) = 3\log 5 - \log 2$$

$$x = \frac{3\log 5 - \log 2}{3\log 2 - \log 5}$$ — 1½ marks for algebra

$$x = \frac{3(0.698\ 97) - 0.301\ 03}{3(0.301\ 03) - 0.698\ 97}$$

$$x = 8.798$$ — ½ mark for consistent answer

3 marks

Method 2

$$(3x+1)\log 2 = (x+3)\log 5$$ — 1 mark for applying logarithms

$$(3x+1)(0.301\ 03) = (x+3)(0.698\ 97)$$

$$0.903\ 09x + 0.301\ 03 = 0.698\ 97x + 2.096\ 91$$

$$(0.903\ 09 - 0.699\ 87)x = 2.096\ 91 - 0.301\ 03$$ — 1½ marks for algebra

$$0.204\ 12x = 1.795\ 89$$

$$x = \frac{1.795\ 89}{0.204\ 12}$$

$$x = 8.798$$ — ½ mark for consistent answer

3 marks

Question No. 1

Outcome: D6

Method 3

$$(3x+1)\ln 2 = (x+3)\ln 5$$

1 mark for applying logarithms

$$3x\ln 2 + \ln 2 = x\ln 5 + 3\ln 5$$
$$3x\ln 2 - x\ln 5 = 3\ln 5 - \ln 2$$
$$x(3\ln 2 - \ln 5) = 3\ln 5 - \ln 2$$
$$x = \frac{3\ln 5 - \ln 2}{3\ln 2 - \ln 5}$$
$$x = \frac{3(1.609\ 44) - 0.693\ 15}{3(0.693\ 15) - 1.609\ 44}$$

1½ marks for algebra

$$x = 8.798$$

½ mark for consistent answer

3 marks

Method 4

$$(3x+1)\ln 2 = (x+3)\ln 5$$

1 mark for applying logarithms

$$(3x+1)(0.693\ 15) = (x+3)(1.609\ 44)$$
$$2.079\ 45x + 0.693\ 15 = 1.609\ 44x + 4.828\ 31$$
$$(2.079\ 45 - 1.609\ 44)x = 4.828\ 31 - 0.693\ 15$$
$$x = \frac{4.135\ 16}{0.470\ 01}$$

1½ marks for algebra

$$x = 8.798$$

½ mark for consistent answer

3 marks

Note(s):

- **deduct** ½ mark if parentheses are assumed but not written in step 1
- **deduct** 1 mark if missing parentheses, and miscalculated
- **deduct** ½ mark for incorrect rounding or incorrect decimal places

EXEMPLAR

$2^{3x+1} = 5^{x+3}$

$\log 2^{3x+1} = \log 5^{x+3}$

$(3x+1)\log 2 = x+3\log 5$

$3x\log 2 + \log 2 = x\log 5 + 3\log 5$

$3x\log 2 - x\log 5 = 3\log 5 - \log 2$

$3x - x = \dfrac{3\log 5 - \log 2}{\log 2}$

$2x = 5.966$

$x = \dfrac{5.966}{2} = 2.983$

$x = 2.983$

Mark: **1½ out of 3**

Note(s):

- deducted ½ mark for missing parentheses on the third line
- deducted 1 mark for incorrect algebra on the sixth line

This page was intentionally left blank.

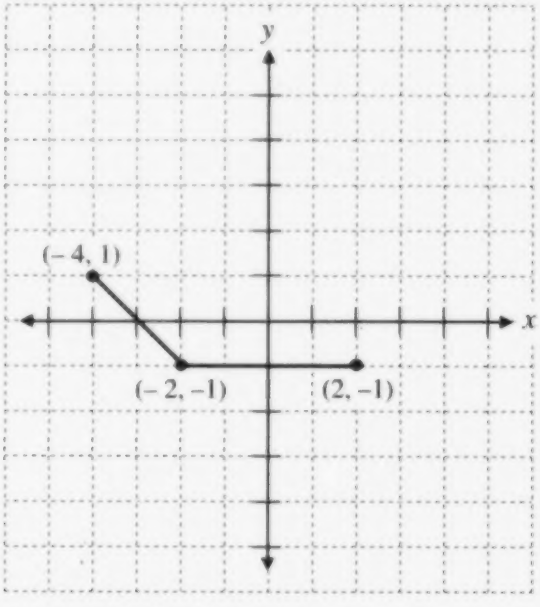
Question No. 2

Outcome: a) B1 b) B4

Given the graph of $y = f(x)$:

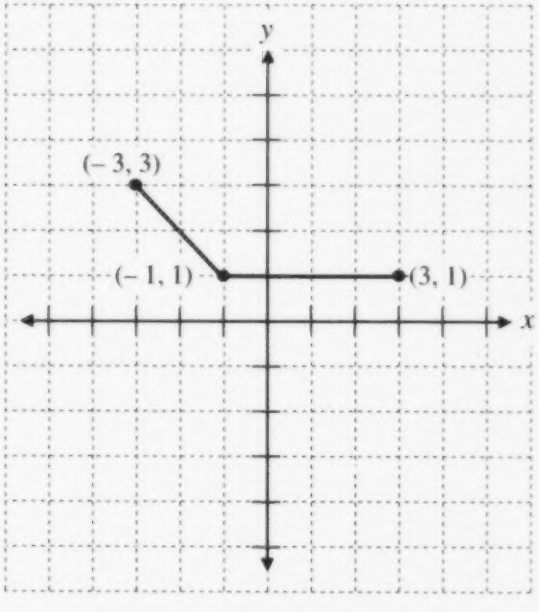
a) Sketch the graph of $y = f(x-1)+2$.

b) Sketch the graph of $y = \dfrac{1}{f(x)}$.



Solution

a)



1 mark for translating one unit to the right

1 mark for translating two units up

2 marks

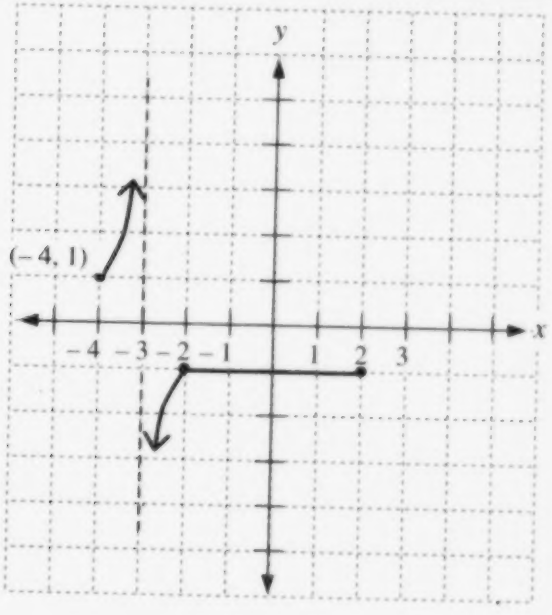
Note(s):

- in a) **deduct** a maximum of ½ mark if arrowheads are included on the graph

Question No. 2

Outcome: a) B1 b) B4

b)



½ mark for asymptote

½ mark for asymptotic behaviour of both branches

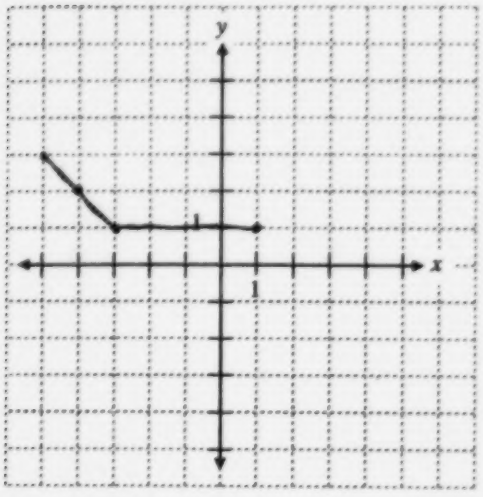
½ mark for horizontal portion of the right branch

½ mark for the endpoints and arrowheads

2 marks

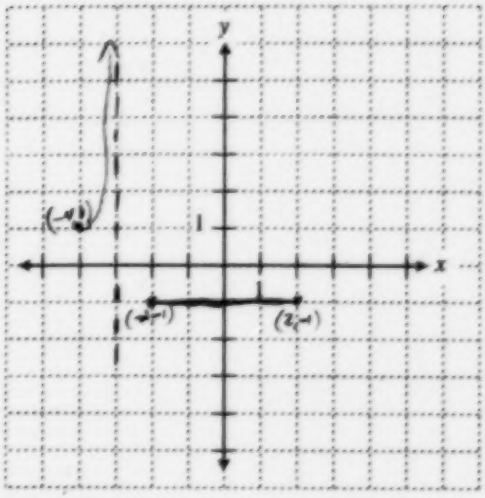
EXEMPLARS

a) Sketch the graph of $y = f(x-1)+2$.



Mark: **1 out of 2**
Note(s):
- wrong horizontal translation

b) Sketch the graph of $y = \dfrac{1}{f(x)}$.



Mark : **1½ out of 2**
Note(s):
- deducted ½ mark for not displaying asymptotic behaviour of the right branch

This page was intentionally left blank.

Question No. 3

Outcome: D6

Solve for x algebraically:

$$\log_2(x-2) + \log_2(x+1) - 2 = 0$$

Solution

$\log_2(x-2) + \log_2(x+1) - 2 = 0$

$\log_2[(x-2)(x+1)] = 2$ — 1 mark for log theorem

$2^2 = (x-2)(x+1)$ — 1 mark for exponential form

$4 = (x-2)(x+1)$

$0 = x^2 - x - 6$

$0 = (x-3)(x+2)$

$x = 3$ ~~$x = -2$~~ — 1 mark for solving the quadratic equation
1 mark for rejection of root

4 marks

EXEMPLAR

$$\log_2(x-2) + \log_2(x+1) - 2 = 0$$

$$\log_2(x-2)(x+1) = 2$$

$$2^2 = (x-2)(x+1)$$

$$4 = x^2 - x - 2$$

$$0 = x^2 - x + 2$$

$$0 = (x-2)(x+1).$$

$x = 2$ ~~$x = -1$~~

Mark: **2 out of 4**

Note(s):

- gave 1 mark for log theorem
- gave 1 mark for exponential form
- no marks for solving the quadratic equation
- no marks for rejection of root, both roots must be rejected for $x = 2$, $x = -1$

Question No. 4

Outcomes: A4, C1

Solve for x where $0 \le x \le 2\pi$.
State the solution(s) correct to 3 decimal places.

$$\sin^2 x + \cos^2 x + \tan^2 x = 3$$

Solution

Method 1

$1 + \tan^2 x = 3$	1 mark for substitution of identity
$\tan^2 x = 2$	
$\tan x = \pm\sqrt{2}$	1 mark for solving for $\tan x$ (½ mark for $\tan x = \sqrt{2}$ and ½ mark for $\tan x = -\sqrt{2}$)
$x = 0.955,\ 2.186,\ 4.097,\ 5.328$	2 marks for values (½ mark for each answer)
	4 marks

Method 2

$1 + \tan^2 x = 3$	1 mark for substitution of identity
$\sec^2 x = 3$	
$\sec x = \pm\sqrt{3}$	
$\cos x = \pm\frac{1}{\sqrt{3}}$	1 mark for solving for $\cos x$ (½ mark for $\cos x = \frac{1}{\sqrt{3}}$ and ½ mark for $\cos x = -\frac{1}{\sqrt{3}}$)
$x = 0.955,\ 2.186,\ 4.097,\ 5.328$	2 marks for values (½ mark for each answer)
	4 marks

Note(s):

- give maximum 3 marks for answers in degrees $x = 54.736°, 125.264°, 234.736°, 305.264°$

EXEMPLAR

$\sin^2 x + \cos^2 x + \tan^2 x = 2$

$1 + \tan^2 x = 2 - 1$

$\tan^2 x = 1$

$\tan x = \frac{\pi}{4}, \frac{5\pi}{4}$

Mark: **1½ out of 4**

Note(s):

- gave 1 mark for substitution of identity only
- gave ½ mark for partially solving for tan x
- did not find values for x: $\tan x = \frac{\pi}{4}, \frac{5\pi}{4}$ is not correct; no marks given

Question No. 5

Outcome: G5

The school indoor soccer team is to consist of 6 girls and 5 boys, chosen at random from a group of 10 girls and 11 boys. Two of the girls and one of the boys are very good friends.

What is the probability that all 3 friends will be on the team?
Briefly explain your calculations.

Solution

Method 1

In addition to the 2 girls, choose 4 from remaining 8 girls.

$$_8C_4 \text{ OR } \binom{8}{4}$$

In addition to the 1 boy, choose 4 from remaining 10 boys.

$$_{10}C_4 \text{ OR } \binom{10}{4}$$

P(all 3 friends are on the team)

$$= \frac{_8C_4 \cdot {}_{10}C_4}{_{10}C_6 \cdot {}_{11}C_5} = \frac{70 \cdot \cancel{210}}{\cancel{210} \cdot 462} = \frac{70}{462} = \frac{5}{33}$$

2 marks for numerator
½ mark for denominator
½ mark for consistent answer

3 marks

OR

$$= \frac{\binom{8}{4}\binom{10}{4}}{\binom{10}{6}\binom{11}{5}}$$

$$= \frac{14\,700}{97\,020} = \frac{5}{33}$$

2 marks for numerator

½ mark for denominator

½ mark for consistent answer

3 marks

Question No. 5

Outcome: G5

Method 2

P(1st girl is on the team) • P(2nd girl is on the team) • P(the boy is on the team)

$= \frac{6}{10} \cdot \frac{5}{9} \cdot \frac{5}{11}$

$= \frac{5}{33}$

1½ marks (½ mark for each fraction)

1 mark for multiplying

½ mark for consistent answer

3 marks

Method 3

P(two girls are on the team) • P(the boy is on the team)

$= \frac{{}_6P_2}{{}_{10}P_2} \cdot \frac{{}_5P_1}{{}_{11}P_1}$

$= \frac{6 \cdot 5}{10 \cdot 9} \cdot \frac{5}{11}$

$= \frac{5}{33}$

2 marks for numerator

½ mark for denominator

½ mark for consistent answer

3 marks

Method 4

P(two girls are on the team) • P(the boy is on the team)

$= \frac{{}_6C_2}{{}_{10}C_2} \cdot \frac{{}_5C_1}{{}_{11}C_1} = \frac{\frac{6!}{2!4!}}{\frac{10!}{2!8!}} \cdot \frac{\frac{5!}{4!1!}}{\frac{11!}{10!1!}} = \frac{6 \cdot 5}{10 \cdot 9} \cdot \frac{5}{11} = \frac{5}{33}$

2 marks for numerator

½ mark for denominator

½ mark for consistent answer

3 marks

Note(s):

- **deduct** 1 mark for conceptual error if the probabilities are added
- final answer may be expressed as a fraction, a decimal $\left(0.15\overline{15} \text{ or } 0.152\right)$ or a percent $\left(15.\overline{15}\% \text{ or } 15.2\%\right)$
- **deduct** ½ mark if explanation of calculations is not given

EXEMPLAR

$$\frac{{}_{10}C_2 \cdot {}_{11}C_1}{{}_{10}C_6 \cdot {}_{11}C_5}$$

$$\frac{45 \cdot 11}{210 \cdot 462} = \frac{495}{97020}$$

$$= 0.005 \text{ ways}$$

Mark: ½ out of 3

Note(s):

- gave ½ mark for the denominator
- gave ½ mark for the consistent answer
- deducted ½ mark for no explanation of calculations given

This page was intentionally left blank.

Question No. 6

Outcome: E4

Find the middle term of the expansion of $(3a^2 - 2b)^8$.

Simplify your answer.

Solution

${}_8C_4\left(3a^2\right)^4\left(-2b\right)^4$ OR $\binom{8}{4}\left(3a^2\right)^4\left(-2b\right)^4$	1½ marks for setup (½ mark for each factor)
$= 70\left(81a^8\right)\left(16b^4\right)$	1 mark for simplification
$= 90\,720\,a^8\,b^4$	½ mark for consistent answer
	3 marks

Note(s):

- give ½ mark for only stating that the fifth term (t_5) is the middle term
- if student finds term four, give maximum 2 marks

  $t_4 = {}_8C_3\left(3a^2\right)^5\left(-2b\right)^3 = -108\,864\,a^{10}\,b^3$
- if second step is omitted but final answer is correct, give full marks

EXEMPLAR

9 terms middle is 5th term

$${}_8C_{5-1}(3a^2)(-2b)$$

$$= 4$$

$${}_8C_4(3a^2)^2(-2b)^2$$

$$70(9a^4)(-4b^2)$$

$$= -2520a^4b^2$$

Mark: 1½ out of 3

Note(s):

- gave ½ mark for ${}_8C_4$
- gave 1 mark for simplification
- deducted ½ mark for arithmetic error: $(-2)^2 \neq -4$
- gave ½ mark for consistent answer

Question No. 7

Outcome: E1

How many four-digit positive integers greater than 4 700 can be formed using the digits 1, 4, 5, 8, 9 if repetition is not allowed?

Briefly explain your calculations.

Solution

Method 1

Case 1: Four-digit number and 4 as first digit

$$\underset{\{4\}}{1} \cdot \underset{\{8,9\}}{2} \cdot \underline{\quad 3 \quad} \cdot \underline{\quad 2 \quad} = 12$$

1 mark for Case 1

Case 2: Four-digit number and 5, 8, or 9 as first digit

$$\underset{\{5,8,9\}}{3} \cdot \underline{\quad 4 \quad} \cdot \underline{\quad 3 \quad} \cdot \underline{\quad 2 \quad} = 72$$

1 mark for Case 2

Total number of four-digit numbers greater than 4 700 is:

$12 + 72 = 84$

1 mark for addition of cases and consistent answer

3 marks

Method 2

Case 1: $\underset{\{4,\ 5,\ 8,\ 9\}}{4} \cdot \underline{\quad 4 \quad} \cdot \underline{\quad 3 \quad} \cdot \underline{\quad 2 \quad} = 96$

1 mark for numbers greater than 4 000

Case 2: $\underset{\{4\}}{1} \cdot \underset{\{1,\ 5\}}{2} \cdot \underline{\quad 3 \quad} \cdot \underline{\quad 2 \quad} = 12$

1 mark for numbers to be excluded

Total number of four-digit numbers greater than 4 700 is:

$96 - 12 = 84$

1 mark for subtraction of cases and consistent answer

3 marks

Question No. 7

Outcome: E1

Method 3

Case 1: Four-digit numbers less than 4 700 starting with 4

$$\frac{1}{\{4\}} \cdot \frac{2}{\{5, 1\}} \cdot \frac{3}{\quad} \cdot \frac{2}{\quad} = 12$$

1 mark for Case 1

Case 2: Four-digit numbers less than 4 700 starting with 1

$$\frac{1}{\{1\}} \cdot \frac{4}{\quad} \cdot \frac{3}{\quad} \cdot \frac{2}{\quad} = 24$$

½ mark for Case 2

$\therefore$ Total number of four-digit numbers less than 4 700 is

$12 + 24 = 36$

Total number of four-digit numbers with no restriction

$$\frac{5}{\quad} \cdot \frac{4}{\quad} \cdot \frac{3}{\quad} \cdot \frac{2}{\quad} = 120$$

½ mark for total number

Therefore the total number of four-digit numbers greater than 4 700 is

$120 - 36 = 84$

1 mark for subtraction

3 marks

Note(s):

- **deduct** ½ mark if explanation of calculations is not given

EXEMPLAR

(a) 4^c $1 \cdot 2 \cdot 3 \cdot 3$ 12

(b) 5^3 $3 \cdot 4 \cdot 3 \cdot 2$ + 72

84

Mark: **2½ out of 3**

Note(s):

- deducted ½ mark for not clearly identifying the cases

This page was intentionally left blank.

Question No. 8

Outcome: G2

Three boxes contain red marbles and blue marbles as shown below.

Box 1	Box 2	Box 3
2 red 4 blue	5 red 4 blue	5 red 1 blue

Jack throws a standard six-sided die to choose a box.

If the die shows 1 or 2 he chooses Box 1.

If the die shows 3, 4 or 5 he chooses Box 2.

If the die shows 6 he chooses Box 3.

He then chooses a marble at random from the box which he selected.

Find the probability that the marble is red.

Solution

Method 1

P(red marble)

$= P(\text{Box 1 and red}) + P(\text{Box 2 and red}) + P(\text{Box 3 and red})$

$= \frac{1}{3}\left(\frac{2}{6}\right) + \frac{1}{2}\left(\frac{5}{9}\right) + \frac{1}{6}\left(\frac{5}{6}\right)$ 3 marks (½ mark for each fraction in product)

$= \frac{1}{9} + \frac{5}{18} + \frac{5}{36}$

$= \frac{19}{36}$ OR $0.52\overline{7}$ 1 mark for adding and consistent answer

4 marks

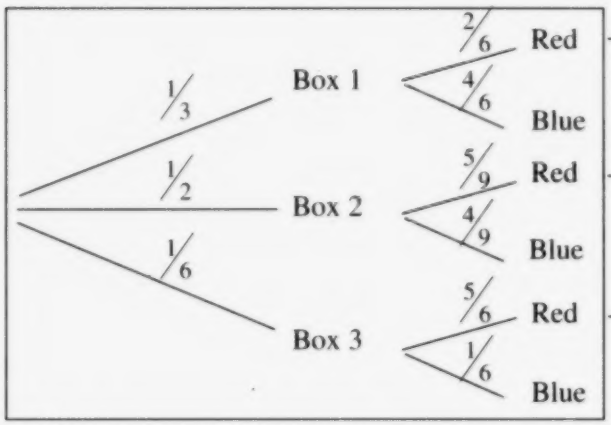
Note(s):

- give a maximum of 3 marks if the problem is solved as conditional probability, e.g.: $P(R/B_1) = \frac{4}{19}$

Question No. 8

Outcome: G2

Method 2



$\left(\frac{1}{3}\right)\left(\frac{2}{6}\right)$ ½ mark for each fraction in product

$\left(\frac{1}{2}\right)\left(\frac{5}{9}\right)$ ½ mark for each fraction in product

$\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)$ ½ mark for each fraction in product

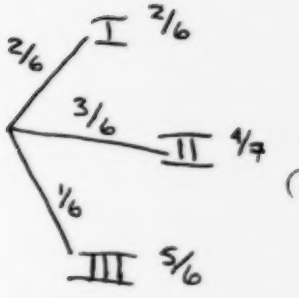
$\frac{1}{9}+\frac{5}{18}+\frac{5}{36}=\frac{19}{36}$ 1 mark for adding and consistent answer

$P(\text{red})=\frac{19}{36}$ 4 marks

Note(s):

- give 1 mark for correctly completed tree diagram with no other correct work

EXEMPLAR



$$\frac{2}{6}\left(\frac{2}{6}\right)+\frac{3}{6}\left(\frac{4}{7}\right)+\frac{1}{6}\left(\frac{5}{6}\right)$$

$$= \frac{4}{12}+\frac{12}{42}+\frac{5}{36} \quad (21)\ (7)$$

$$= \frac{84+72+35}{252}$$

$$= \frac{191}{252}$$

Mark: 3 out of 4

Note(s):

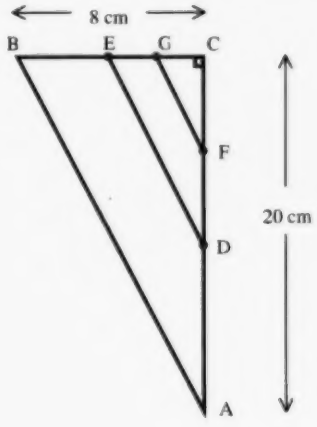
- deducted ½ mark for an incorrect fraction in the middle product
- deducted ½ mark for an arithmetic error i.e., $\left(\frac{2}{6}\right)\left(\frac{2}{6}\right)\neq\left(\frac{4}{12}\right)$

This page was intentionally left blank.

Question No. 9

Outcome: H3

ΔABC is a right triangle with $\overline{BC}$ having a length of 8 cm, and $\overline{AC}$ having a length of 20 cm. New right triangles are formed by joining the midpoints of the perpendicular sides and retaining the right angles as shown in the diagram ($\Delta ABC, \Delta DEC, \Delta FGC$ and so on).
This process continues an infinite number of times.
Find the sum of the areas of these triangles in cm^2.



Solution

Series is: $80 + 20 + 5 + \ldots$

½ mark for correct series (or identifying $t_1 = 80$)

$$S_\infty = \frac{t_1}{1-r}$$

1 mark for $r = \frac{1}{4}$

$$= \frac{80}{1-\frac{1}{4}}$$

½ mark for substitution into correct formula

$$= \frac{80}{\frac{3}{4}}$$

$$= \frac{320}{3}$$

$$= 106\frac{2}{3}\ cm^2 \quad \text{OR} \quad \frac{320}{3}\ cm^2$$

1 mark for consistent answer

3 marks

Note(s):

- give 2 marks if done as a partial sum and exact correct answer,
  e.g.: $106\frac{2}{3}\ cm^2$ OR $106.\overline{6}\ cm^2$ OR $\frac{320}{3}\ cm^2$
- give 1 mark if done as a partial sum and approximate answer, e.g.: $106.6\ cm^2$ OR $106.67\ cm^2$

EXEMPLAR

$A = b * h \div 2$

$A: 8 * 20 \div 2$

$A: 80$

$A = 6 * h \div 2$

$A = 4 * 20 \div 2$

$A = 40$

$S\infty = \frac{t_1}{1-r}$

$S\infty = \frac{80}{1-\frac{1}{2}}$

$S\infty = \frac{80}{\frac{1}{2}} = 160$

Mark: **2 out of 3**
Note(s):
- deducted 1 mark for incorrect *r* value

Question No. 10

Outcome: C2

a) Prove the following identity: $\dfrac{1+\cos 2x}{\sin 2x}=\cot x$.

b) Give one value of x where $\dfrac{1+\cos 2x}{\sin 2x}$ is undefined.

Solution

Method 1

a) $\text{LHS}=\dfrac{1+\cos 2x}{\sin 2x}$

$=\dfrac{1+\left(2\cos^2 x-1\right)}{2\sin x\cos x}$ — 1 mark for identity; ½ mark for identity

$=\dfrac{\cancel{2}\cos^2 x}{\cancel{2}\sin x\cos x}$ — 1 mark for simplifying

$=\dfrac{\cos x}{\sin x}$

$=\cot x=\text{RHS}$ — ½ mark for identity

3 marks

Method 2

$\text{LHS}=\dfrac{1+\cos 2x}{\sin 2x}$

$=\dfrac{1+\left(\cos^2 x-\sin^2 x\right)}{2\sin x\cos x}$ — 1 mark for identity; ½ mark for identity

$=\dfrac{\left(1-\sin^2 x\right)+\cos^2 x}{2\sin x\cos x}$ — ½ mark for identity

$=\dfrac{\cancel{2}\cos^2 x}{\cancel{2}\sin x\cos x}$

$=\dfrac{\cos x}{\sin x}$ — ½ mark for simplifying

$=\cot x=\text{RHS}$ — ½ mark for identity

3 marks

Question No. 10

Outcome: C2

Method 3

$$\text{LHS} = \frac{1+\cos 2x}{\sin 2x}$$

$$= \frac{1+\left(1-2\sin^2 x\right)}{\sin 2x}$$ 1 mark for identity

$$= \frac{2-2\sin^2 x}{2\sin x\cos x}$$ ½ mark for identity

$$= \frac{1-\sin^2 x}{\sin x\cos x}$$ ½ mark for simplifying

$$= \frac{\cos^2 x}{\sin x\cos x}$$ ½ mark for identity

$$= \frac{\cos x}{\sin x}$$

$$= \cot x = \text{RHS}$$ ½ mark for identity

3 marks

b) $x = 0$ or any integral multiple of $\frac{\pi}{2}$

1 mark

Note(s):

- give ½ mark if the student indicates when the expression is undefined by stating either:
  $\sin 2x \neq 0$ (expression is defined)
  $\sin 2x = 0$ (expression is undefined)

EXEMPLARS

a)

RHS $\dfrac{1+1-2\cos^2 x}{2\sin x\cos x}$

$\dfrac{2-2\cos^2 x}{2\sin x\cos x}$

$\dfrac{2(1-\cos^2 x)}{2\sin x\cos x}$

$\dfrac{\sin^2 x}{\sin x\cos x} = \cot x = LHS$

Mark: 1½ out of 3
Note(s):
- ½ mark for $2\sin x\cos x$
- 1 mark for simplifying

b)

$\sin 2x \neq 0$

$2x \neq \pi/2$

$x \neq \pi/4$

$\pi/4$

Mark: 0 out of 1

Part 2—Restricted-Response Questions

Question No. 26

Outcome: A1

A 40° angle is equivalent to an angle of _____ radians.

Write your answer as an exact value.

Answer

$\frac{40\pi}{180}$ OR $\frac{2\pi}{9}$ OR $\frac{40°\pi}{180°}$ | 1 mark

Note(s):

- give ½ mark if correct proportion given but incorrect final answer
- **deduct** ½ mark if only one degree symbol left on final answer

Question No. 27

Outcome: A3, A6

The exact value of $\sin^{-1}\left[\sin\left(\frac{5\pi}{6}\right)\right]$ is:

Answer

$\frac{\pi}{6}$ OR 30° | 1 mark |

Note(s):

- give ½ mark if $\sin\frac{5\pi}{6} = \frac{1}{2}$ is shown
- give no marks for an answer of $\frac{5\pi}{6}$ or multiple answers

Question No. 28

Outcome: C2, A3

Find the exact value of $2\cos^2\left(\frac{\pi}{8}\right)-1$.

Answer

$\cos\left(2\left(\frac{\pi}{8}\right)\right)$

$= \cos\left(\frac{\pi}{4}\right)$

} ½ mark

$= \frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$ ½ mark

1 mark

Question No. 29

Outcome: B4, A6

Write an equation for one vertical asymptote of the graph of $y = \sec x$.

Answer

Any one of:

$x = -\frac{3\pi}{2}$
OR
$x = -\frac{\pi}{2}$
OR
$x = \frac{\pi}{2}$
OR
$x = \frac{3\pi}{2}$
.
.
.
etc.

$x = -270°$
OR
$x = -90°$
OR
$x = 90°$
OR
$x = 270°$
.
.
.
etc.

1 mark

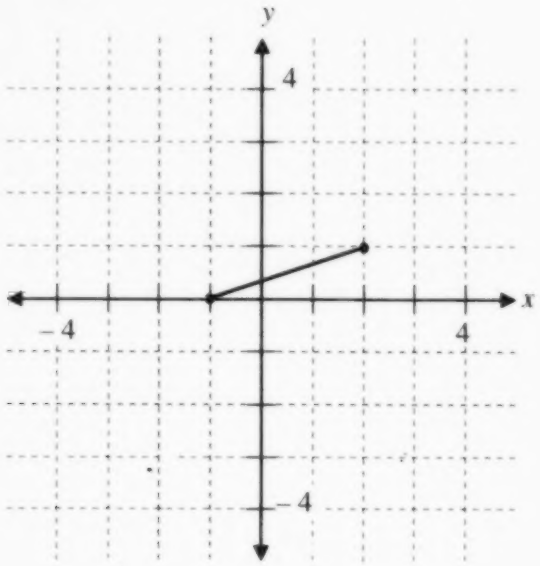
Note(s):

- give ½ mark if written as $x \neq -\frac{3\pi}{2}$ etc.
- give no marks if answer is not in equation form

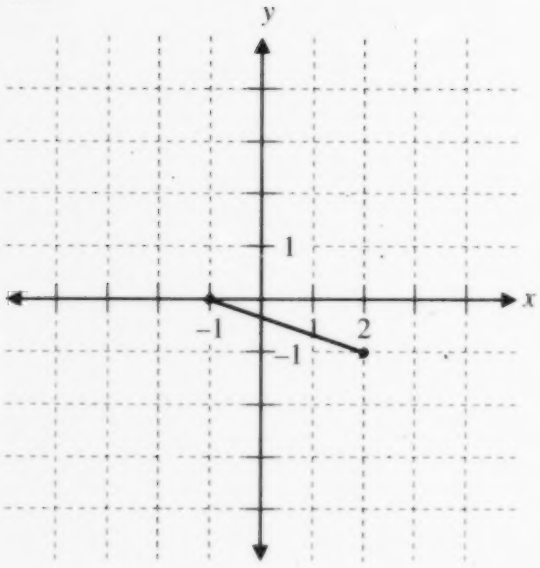
Question No. 30

Outcome: B3

The graph of the function $y = g(x)$ is shown on the left.
On the grid at right, sketch a clearly labelled graph of $y = -g(x)$.



Answer



1 mark

Note(s):

- give no marks for $y = g(-x)$

Question No. 31

Outcome: D2

Solve for x:

$$2^{3x-1} = 4^{x+1}$$

Answer

$$2^{3x-1} = \left(2^2\right)^{x+1}$$

$$3x - 1 = 2x + 2 \qquad \boxed{1 \text{ mark}}$$

$$x = 3$$

Note(s):

- give ½ mark for correct expressions with a common base for example:

  $2^{3x-1} = \left(2^2\right)^{x+1}$

  OR

  $2^{3x-1} = 2^{2x+2}$

Question No. 32

Outcome: D6

Given $f(x) = \log(x+10)+3$, evaluate $f(0)$.

Answer

$f(0) = \log(0+10) + 3$

$= 1 + 3$

$= 4$

1 mark

Note(s):

- give no marks for just substituting 0 for x,
  e.g.: $f(0) = \log 10 + 3$ OR $f(0) = \log(0+10)+3$
- give ½ mark for evaluating $\log 10 = 1$

Question No. 33

Outcome: E1

In how many ways can all the letters of the word T A B L E be arranged if the vowels and consonants must alternate?

Answer

12 OR 3!2! OR $\frac{3}{C} \cdot \frac{2}{V} \cdot \frac{2}{C} \cdot \frac{1}{V} \cdot \frac{1}{C}$ 1 mark

or the following list:

TABEL	TALEB
TEBAL	TELAB
BATEL	BALET
BETAL	BELAT
LATEB	LABET
LETAB	LEBAT

Question No. 34

Outcome: E3

Simplify:

$$\frac{{}_{500}C_{400}}{{}_{500}C_{100}}$$

Answer

$$= \frac{\frac{500!}{400!\ 100!}}{\frac{500!}{100!\ 400!}}$$

1 mark

$$= 1$$

Note(s):

- give ½ mark for correct substitution in formula

Question No. 35

Outcome: F3

An ellipse, centred at the origin, has x-intercepts of 3 and –3 and y-intercepts of 4 and –4.
Write its equation.

Answer

$\frac{x^2}{9} + \frac{y^2}{16} = 1$

OR

$16x^2 + 9y^2 = 144$

1 mark

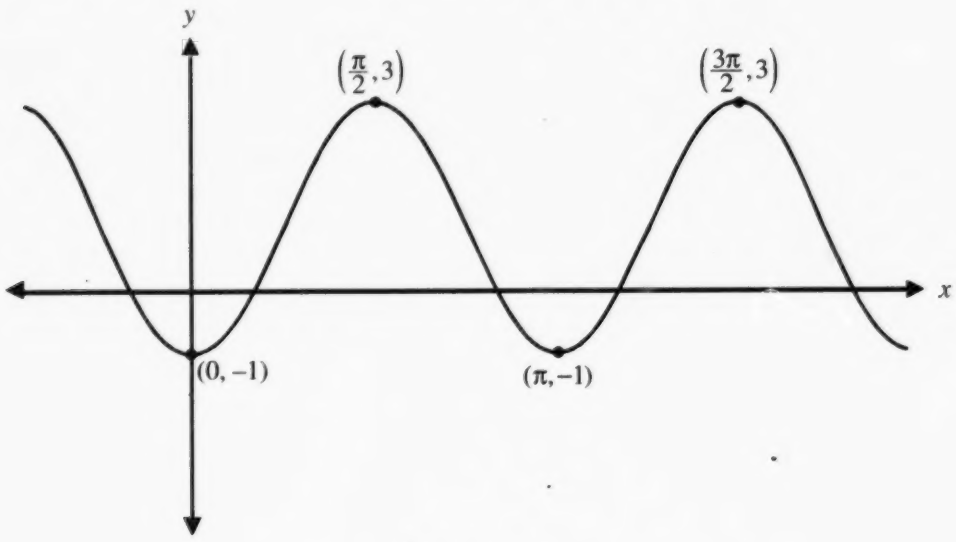
Note(s):

- other equivalent forms are acceptable
- give ½ mark for: $\frac{x^2}{3} + \frac{y^2}{16} = 1$ OR $\frac{x^2}{9} + \frac{y^2}{4} = 1$ OR $16x^2 + 3y^2 = 48$ OR $4x^2 + 9y^2 = 36$
- give ½ mark for incorrectly simplifying $\frac{x^2}{9} + \frac{y^2}{16} = 1$ to $16x^2 + 9y^2 = 1$

Question No. 36

Outcome: B7

What is the amplitude of the function sketched below?



Answer

2 | 1 mark

Note(s):

- give no marks for an answer of 4

Question No. 37

Outcome: D4

Find the value of the expression:

$$(\log_3(\log_5 125))$$

Answer

$\log_3(\log_5 125)$

$= \log_3 3$ 1 mark

$= 1$

Note(s):

- give ½ mark for $\log_5 125 = 3$

Question No. 38

Outcome: D4

The point (27, 3) lies on the graph of $y = \log_b x$.
The exact value of b is:

Answer

$3 = \log_b 27$

$b^3 = 27$ 1 mark

$b = 3$

Note(s):

- give ½ mark for substitution of point
- give ½ mark if student incorrectly substitutes values but correctly changes equation to exponential form

Question No. 39

Outcome: E1

Using **all** the letters in the word E X C E L L E N T, indicate the number of possible distinguishable arrangements that can be made.
Leave your answer in factorial form.

Answer

$\frac{9!}{3!2!}$

1 mark

Question No. 40

Outcome: E4

State the position of the term containing x^{30} in the expansion of $\left(2x^3+4\right)^{12}$.

Answer

Method 1

$\left(2x^3\right)^{12}(4)^0;\ \left(2x^3\right)^{11}(4)^1;\ \left(2x^3\right)^{10}(4)^2$

$x^{36},\ x^{33},\ x^{30}$

3^{rd} term

1 mark

Method 2

$t_{k+1} = {}_{12}C_k\left(2x^3\right)^{12-k}(4)^k$

Consider only "x" terms:

$\left(x^3\right)^{12-k} = x^{30}$

$36-3k=30$

$-3k=-6$

$k=2$

the position of the term is $k+1$ or 3^{rd} term

1 mark

Note(s):

- give ½ mark for establishing a correct pattern of exponents

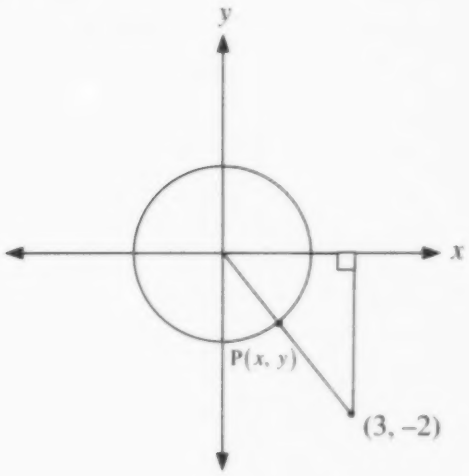
Part 2—Open-Response Questions

Question No. 41

Outcome: A2

P is the point of intersection of the unit circle and the line segment joining the origin and the point (3, –2). Find the coordinates for P.

Solution



$r^2 = (3)^2 + (-2)^2$
$r^2 = 9 + 4$
$r^2 = 13$
$r = \sqrt{13}$

} 1 mark for calculating r

$P = \left(\frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right)$

1 mark for the correct ratios
1 mark for the correct signs

3 marks

Note(s):

- **deduct** 1 mark if the x and y coordinates are reversed in the ordered pair
- answer need not be given as an ordered pair to receive full marks
- **deduct** 1 mark for indicating the wrong quadrant

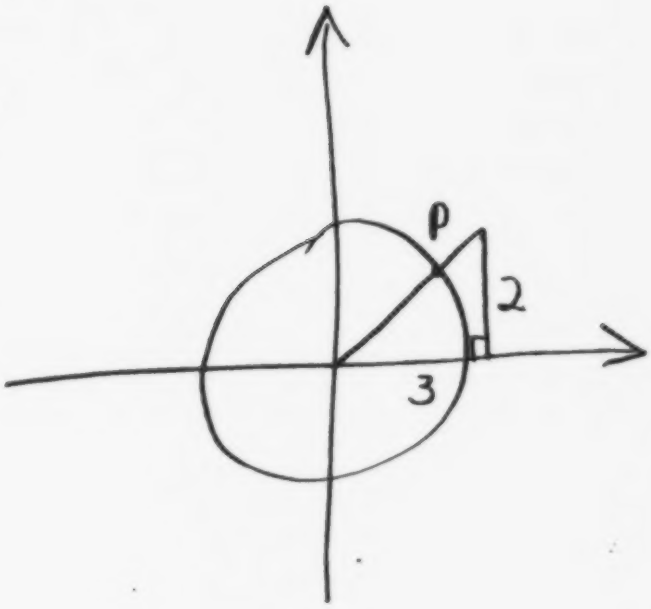
EXEMPLAR

$r^2 = (3)^2 + (2)^2$

$r^2 = 9 + 4$

$r = \sqrt{13}$

$P = \left(\frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}}\right)$



Mark: **2 out of 3**

Note(s):

- deducted 1 mark for wrong quadrant

Question No. 42

Outcomes: F3, B1

The graph of the conic section whose equation is $\frac{x^2}{4} + \frac{y^2}{1} = 1$ is translated 3 units to the right.

a) Write an equation of the new conic section.

b) State the domain of the new conic section.

c) State the range of the new conic section.

Solution

a) $\frac{(x-3)^2}{4} + \frac{y^2}{1} = 1$ — 1 mark

b) $[1, 5]$ — 1 mark

c) $[-1, 1]$ — 1 mark

Note(s):

- **deduct** maximum ½ mark for incorrect brackets e.g.: $(1, 5)$ OR $(1, 5]$ OR $[1, 5)$ in either b) or c)

EXEMPLARS

a)

$$\frac{(x+3)^2}{4} + \frac{y^2}{1} = 1$$

Mark: 0 out of 1

b)

$(-5, -1)$

Mark: ½ out of 1
Note(s):
- gave 1 mark for answer consistent with equation given in a)
- deducted ½ mark for incorrect brackets

c)

$(-1, 1)$

Mark: 1 out of 1
Note(s):
- no deduction for incorrect brackets because it was deducted in b)

Question No. 43

Outcome: A2

If $\sec\theta = -3$ and $\tan\theta > 0$, find the value of $\sin\theta$.

Solution

Method 1

$$\cos\theta = \frac{-1}{3}$$ ½ mark for reciprocal identity

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta + \left(\frac{-1}{3}\right)^2 = 1$$ ½ mark for substitution

$$\sin^2\theta + \frac{1}{9} = 1$$

$$\sin^2\theta = \frac{8}{9}$$

$$\sin\theta = \pm\frac{\sqrt{8}}{3}$$ 1 mark for solving for $\sin\theta$

since $\cos\theta < 0$ and $\tan\theta > 0$, in Quad. III

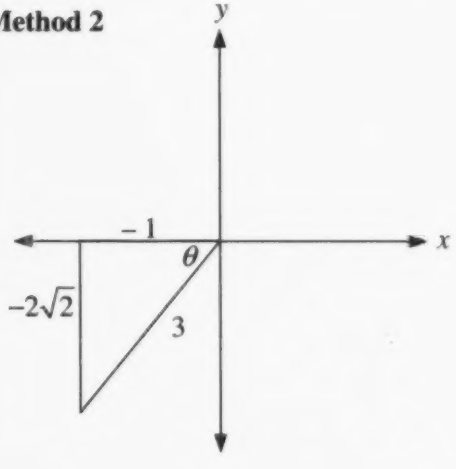
then $\sin\theta = \frac{-\sqrt{8}}{3}$

OR $\sin\theta = \frac{-2\sqrt{2}}{3}$

} 1 mark for correct quadrant

3 marks

Method 2



1 mark for the correct quadrant or the correct sign of trig ratio

1 mark for Pythagorean theorem

1 mark for the correct $\sin\theta$ ratio

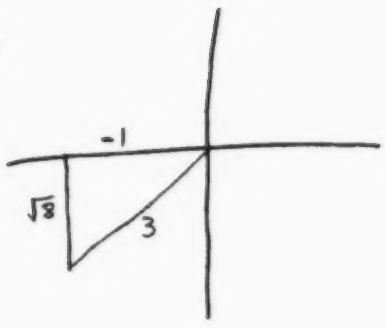
$$\sin\theta = -\frac{2\sqrt{2}}{3}$$

3 marks

EXEMPLAR

$\frac{1}{\cos}$ −

$\cos\theta = \frac{-1}{3}$



$\sin\theta = \sqrt{8}$

$= 2\sqrt{2}$

Mark: **1½ out of 3**

Note(s):

- gave 1 mark for Pythagorean theorem
- gave ½ mark for quadrant (should be $-\sqrt{8}$)

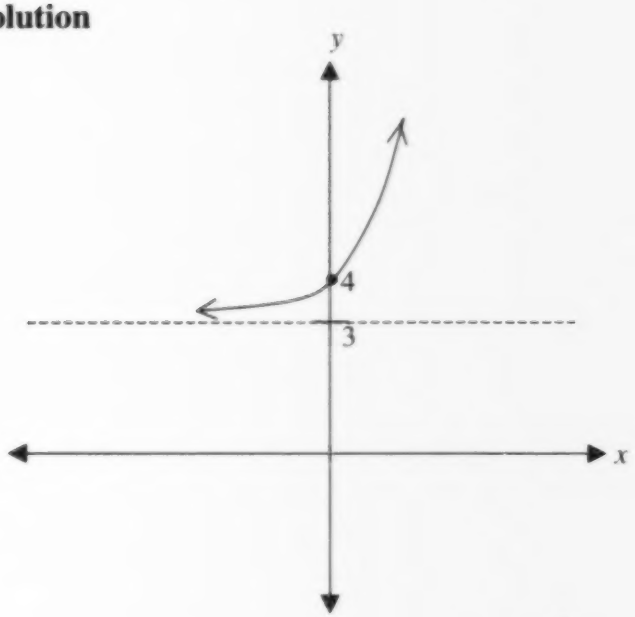
Question No. 44

Outcomes: D7

Sketch a clearly labelled graph of the following function.
State the y-intercept and the equation(s) of any asymptotes.

$$y = e^x + 3$$

Solution



1 mark for correct graph

y-intercept ___4___ 1 mark for consistent answer

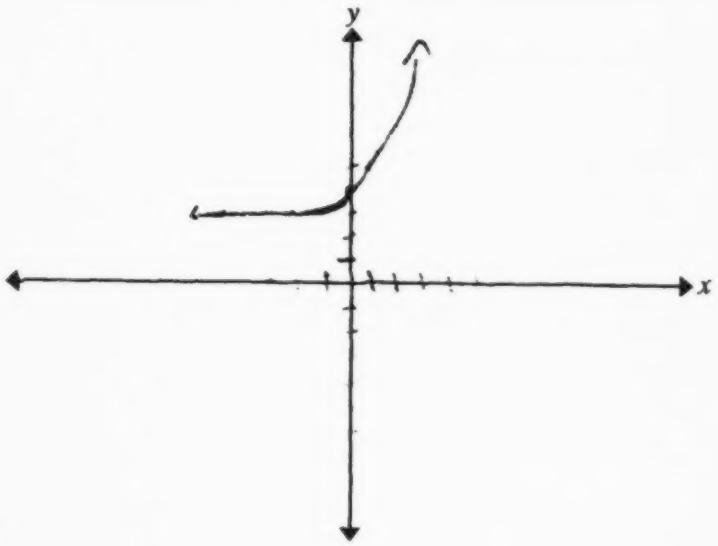
asymptote(s) ___$y = 3$___ 1 mark for consistent answer

3 marks

Note(s):

- correct graph must be an increasing exponential function with intercept and asymptote labelled
- **deduct** ½ mark if y-intercept is given as (0, 4) in the space provided
- give ½ mark if asymptote given as $y \neq 3$
- give 0 marks if asymptote given as 3

EXEMPLAR



y-intercept (0, 4)

asymptote(s) $y = 3$

Mark: 2 out of 3
Note(s):

- deducted ½ mark for *y*-intercept given incorrectly as ordered pair instead of 4
- deducted ½ mark for asymptote missing on graph

Question No. 45

Outcome: D4

Given $f(x) = \log_2(x-3)$, find an equation for $f^{-1}(x)$.

Solution

$f(x)$: $y = \log_2(x-3)$

$f^{-1}(x)$: $x = \log_2(y-3)$ — 1 mark for switching x and y

$2^x = y-3$ — 1 mark for exponential form

$y = 2^x + 3$ — ½ mark for solving for y

$\therefore f^{-1}(x) = 2^x + 3$ — ½ mark for correct inverse function notation

3 marks

EXEMPLAR

$y = \log_2(x-3)$

$x = \log_2(y-3)$

$\frac{x+3}{\log_2} = y$

Mark: **1 out of 3**

Note(s):

- gave 1 mark for switching x and y variables

Question No. 46

Outcome: G2

The probability of the chemistry teacher giving a test next Tuesday is 0.2 and the probability of the mathematics teacher giving a test next Tuesday is 0.4.

a) Find the probability that next Tuesday there will be a test on chemistry and a test on mathematics.

b) Find the probability that next Tuesday there will be a test on chemistry or a test on mathematics.

Solution

a) $P(\text{chemistry \& math}) = (0.2)(0.4)$

$= 0.08$

1 mark

b) $P(\text{chemistry or math}) = 0.2 + 0.4 - 0.08$

$= 0.52$

1 mark for substitution into the formula

1 mark for consistent answer

2 marks

Note(s):

- give a maximum of ½ mark if only addition is performed in part b) (0.2 + 0.4 = 0.6)

EXEMPLARS

a)

$$\frac{2}{10} \cdot \frac{4}{10} = \frac{8}{20} = \frac{2}{5}$$

Mark: ½ out of 1
Note(s):
- arithmetic error

b)

$$\frac{2}{10} + \frac{4}{10} = \frac{6}{10} = \frac{3}{5}$$

Mark: ½ out of 2
Note(s):
- gave ½ mark for addition only

Question No. 47

Outcomes: F2, F3

Determine the coordinates of the vertices of the conic section with the following equation:

$$9x^2 - 4y^2 - 8y - 40 = 0$$

Solution

$$9x^2 - 4\left(y^2 + 2y\right) = 40$$

$$9x^2 - 4\left(y^2 + 2y + 1\right) = 40 - 4$$

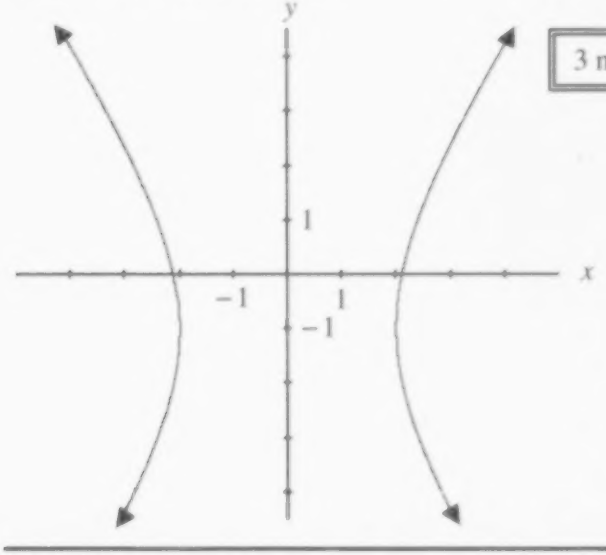
$$9x^2 - 4(y+1)^2 = 36$$

$$\frac{x^2}{4} - \frac{(y+1)^2}{9} = 1$$

1 mark for completing the square
(½ mark for left hand side, ½ mark for right hand side)

Vertices (2, – 1) and (–2, –1)

2 marks (½ mark for each coordinate)



3 marks

Note(s):

- give a maximum of 2 marks for correct diagram without labelled coordinates
- mark vertices consistent with standard form of equation

EXEMPLAR

$9x^2 - 4y^2 - 8y = 40$

$9x^2 - 4(y^2 - 2y) = 40$

$9x^2 - 4(y^2 - 2y + 1^2) = 40 + 1^2(4)$

$9x^2 - 4(y-1)^2 = 44$

$\frac{x^2}{44/9} - \frac{(y-1)^2}{11} = 1$

V $\left(\pm\sqrt{\frac{44}{9}}, \pm\sqrt{11}\right)$

Mark: **1 out of 3**
Note(s):
- gave 1 mark for x coordinates of vertices consistent with equation

Question No. 48

Outcome: C1

Express in simplest form:

$$\frac{\left(\cos x + \dfrac{\sin^2 x}{\cos x}\right)}{\sec x}$$

Solution

Method 1

$\left(\cos x + \dfrac{\sin^2 x}{\cos x}\right)\cos x$ — 1 mark for multiplying by reciprocal

$\cos^2 x + \sin^2 x$ — ½ mark for simplifying

$= 1$ — ½ mark for final answer

2 marks

Method 2

$$= \frac{\dfrac{\cos^2 x + \sin^2 x}{\cos x}}{\sec x}$$ — ½ mark for common denominator

$$= \frac{\dfrac{\cos^2 x + \sin^2 x}{\cos x}}{\dfrac{1}{\cos x}}$$ — ½ mark for identity

$$= \frac{1}{\cos x} \cdot \frac{\cos x}{1}$$ — ½ mark for multiplying by reciprocal

$= 1$ — ½ mark for final answer

2 marks

Note(s):

- **deduct** ½ mark if two or more "x" are dropped

EXEMPLAR

$$\frac{\left(\cos x + \dfrac{\sin^2 x}{\cos x}\right)}{\sec x}$$

$$= \left(\cos x + \frac{\sin^2 x}{\cos x}\right) \times \frac{1}{\cos x}$$

$$= \frac{\cos^2 + \sin^2 x}{\cos x} \times \frac{1}{\cos x}$$

$$= \frac{1}{\cos^2 x}$$

Mark: **1 out of 2**

Note(s):

- deducted 1 mark for incorrect operation in line 2

Question No. 49

Outcome: C2

Find the exact value of:

a) $\sin\left(\frac{5\pi}{12}\right)$ b) $\csc\left(\frac{5\pi}{12}\right)$

Solution

Method 1

a) $\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$ ½ mark for combination

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$ 2 marks for function values (½ mark for each correct value)

$= \frac{\sqrt{6} + \sqrt{2}}{4}$ ½ mark for simplification

3 marks

b) $\csc\left(\frac{5\pi}{12}\right) = \frac{1}{\sin\left(\frac{5\pi}{12}\right)}$

therefore $\csc\left(\frac{5\pi}{12}\right) = \frac{4}{\sqrt{6} + \sqrt{2}}$ OR $\left(\sqrt{6} - \sqrt{2}\right)$

1 mark

Method 2

a) $\sin\left(\frac{5\pi}{12}\right) = \sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$ ½ mark for combination

$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$ 2 marks for function values (½ mark for each correct value)

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3} + 1}{2\sqrt{2}}$ ½ mark for simplification

3 marks

b) $\csc\left(\frac{5\pi}{12}\right) = \frac{2\sqrt{2}}{\sqrt{3} + 1}$ OR $\left(\sqrt{6} - \sqrt{2}\right)$

1 mark

Question No. 49

Outcome: C2

Method 3

a)
$$\cos\left(\frac{5\pi}{6}\right) = 1 - 2\sin^2\left(\frac{5\pi}{12}\right)$$
$$2\sin^2\left(\frac{5\pi}{12}\right) = 1 - \cos\frac{5\pi}{6}$$

1 mark for identity

$$= 1 - \left(\frac{-\sqrt{3}}{2}\right)$$

½ mark for substitution

$$= \frac{2 + \sqrt{3}}{2}$$

½ mark for simplifying

$$\sin^2\left(\frac{5\pi}{12}\right) = \frac{2 + \sqrt{3}}{4}$$

½ mark for dividing by 2

$\sin\left(\frac{5\pi}{12}\right) = \frac{\sqrt{2 + \sqrt{3}}}{2}$, positive since $\frac{5\pi}{12}$ terminates in quadrant I

½ mark for appropriate root

3 marks

b) $\csc\left(\frac{5\pi}{12}\right) = \frac{2}{\sqrt{2 + \sqrt{3}}}$

1 mark

Note(s):

- there are many other possible combinations such as:
  $\frac{5\pi}{12} = \left(\frac{3\pi}{4} - \frac{\pi}{3}\right)$
  $\frac{5\pi}{12} = \left(\frac{2\pi}{3} - \frac{\pi}{4}\right)$
  $\frac{5\pi}{12} = \left(\frac{5\pi}{4} - \frac{5\pi}{6}\right)$
- degrees may be used
- **deduct** ½ mark for each incorrect function value

EXEMPLARS

a)

$$\sin \frac{5\pi}{12} = 105^\circ$$

$$105^\circ = 45^\circ + 60^\circ$$

$$\sin 105^\circ = \sin 45^\circ + \sin 60^\circ$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2} + \sqrt{3}}{2}$$

Mark: ½ out of 3
Note(s):
- gave 1 mark for two correct values
- deducted ½ mark for notation error in first line

b)

$$\csc \frac{5\pi}{12} = \frac{2}{\sqrt{2} + \sqrt{3}}$$

Mark: 1 out of 1
Note(s):
- gave 1 mark for answer consistent with answer in a)

This page was intentionally left blank.

Question No. 50

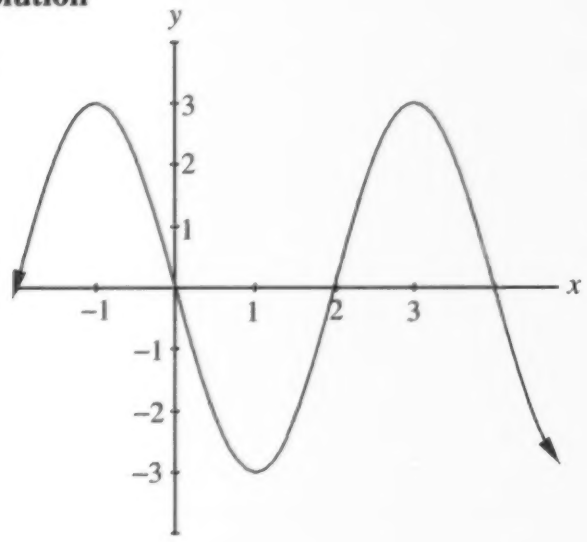
Outcome: a) B7 b) B5

a) Sketch a clearly labelled graph of at least one period of $f(x)=3\cos\left(\frac{\pi}{2}(x+1)\right)$.

b) Sketch $y=|f(x)|$.

Solution

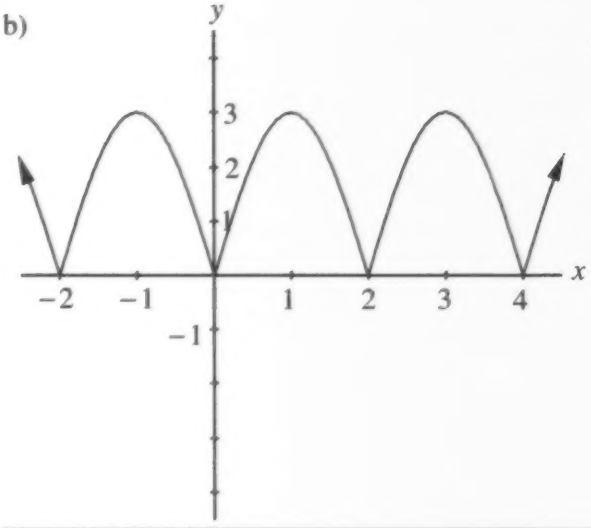
a)



½ mark for general shape
½ mark for amplitude
1 mark for period
1 mark for horizontal shift

3 marks

b)



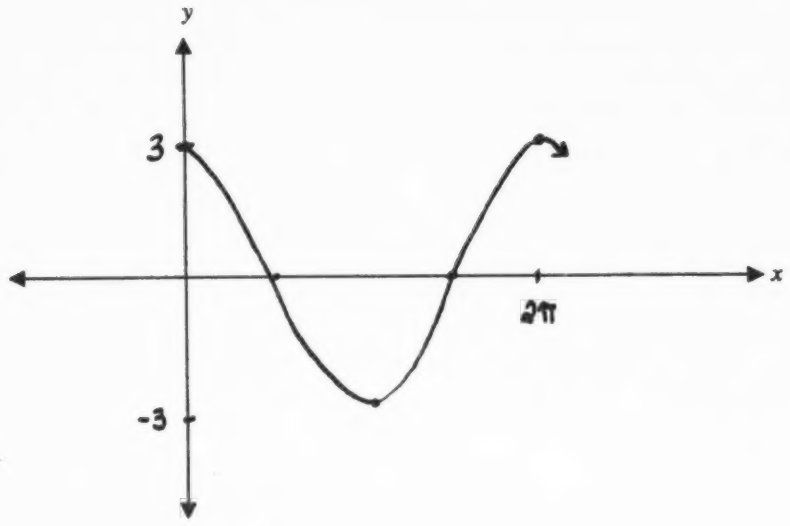
1 mark

Note(s):

- give a maximum of 1½ marks if student correctly calculates the parameters without including a graph

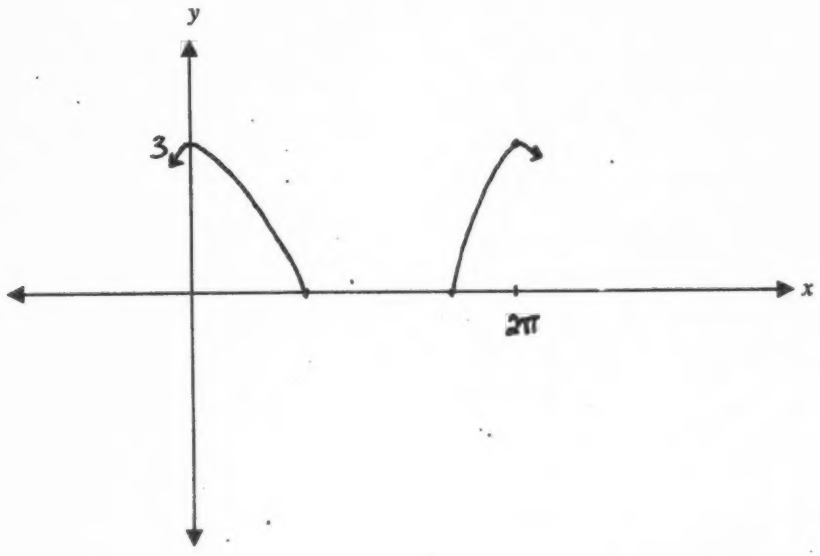
EXEMPLARS

a)



Mark: **1 out of 3**
Note(s):
- gave ½ mark for correct shape
- gave ½ mark for correct amplitude

b)



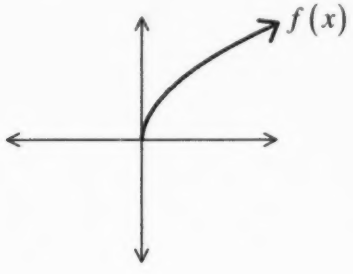
Mark: **0 out of 1**

Question No. 51

Outcome: B3

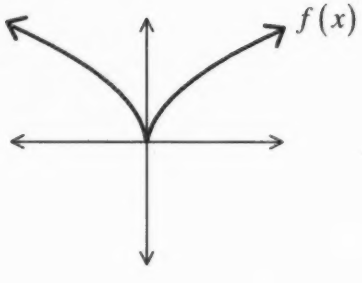
a) Complete the sketch of $f(x)$ below if $f(x)$ is even.

b) Complete the sketch of $f(x)$ below if $f(x)$ is odd.



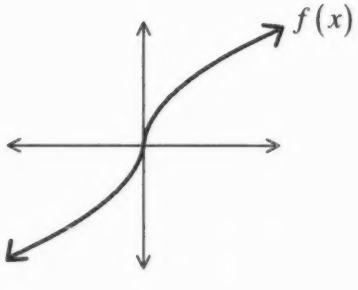
Solution

a)



1 mark

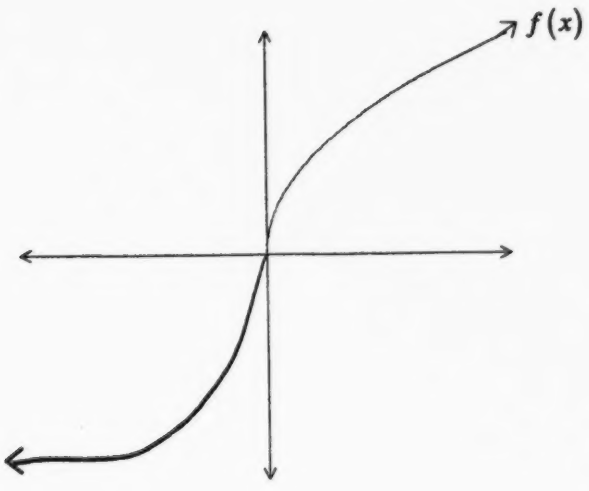
b)



1 mark

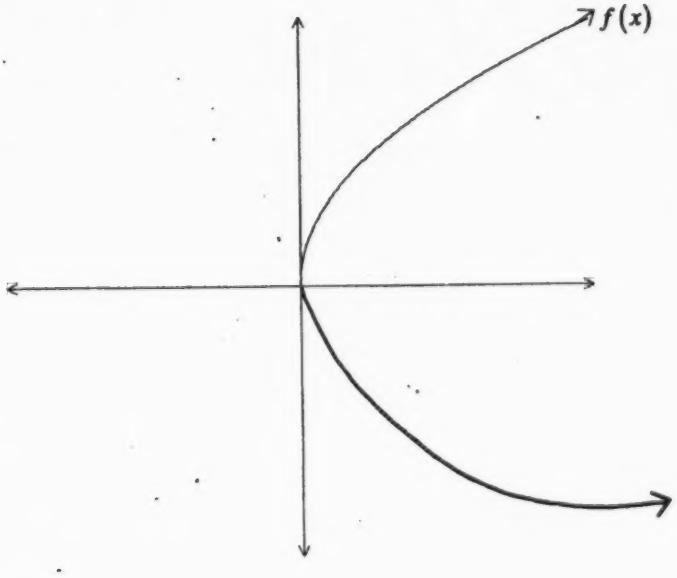
EXEMPLARS

a)



Mark: 0 out of 1

b)

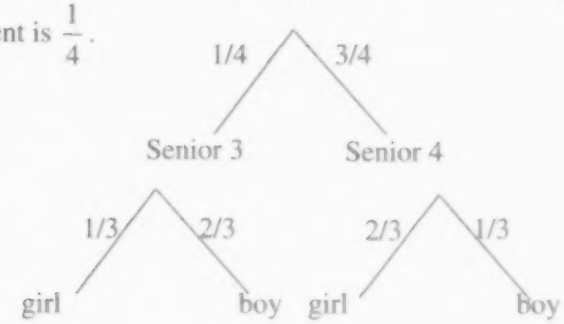


Mark: 0 out of 1

Question No. 52

Outcome: G4

From a group of Senior 3 and Senior 4 students, a principal is going to choose one student at random. The tree diagram below illustrates the probabilities. For example, the probability of selecting a Senior 3 student is $\frac{1}{4}$.



If the principal chooses a girl, what is the probability that the girl is in Senior 3?

Solution

$$P(\text{S3 and girl}) = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}$$

½ mark for numerator

$$P(\text{girl}) = \frac{1}{4} \cdot \frac{1}{3} + \frac{3}{4} \cdot \frac{2}{3}$$

$$= \frac{7}{12}$$

1 mark for denominator

$$P(\text{S3/G}) = \frac{\frac{1}{12}}{\frac{7}{12}} = \frac{1}{12} \cdot \frac{12}{7} = \frac{1}{7}$$

½ mark for consistent answer

2 marks

EXEMPLAR

$$P(A \cap B) = P(A) \cdot P(B|A)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B|A) = \frac{\frac{1}{3} \cdot \frac{2}{3}}{\frac{1}{4} \cdot \frac{1}{3} + \frac{1}{4} \cdot \frac{2}{3}}$$

$$= \frac{\frac{2}{9}}{\frac{3}{12}}$$

$$= \frac{2}{9} \cdot \frac{12}{3} = \frac{24}{27}$$

Mark: ½ out of 2
Note(s):
- gave ½ mark for a consistent answer

This page was intentionally left blank.

Appendices

Appendix A

MARKING GUIDELINES

- Arithmetic error, deduct ½ mark
- Conceptual error, deduct 1 mark
- Stating an intercept as an ordered pair, deduct ½ mark
- Changing an expression to an equation and vice versa, deduct ½ mark
- Missing or incorrect units of measure, deduct ½ mark
- Missing degree symbol in angle measure, deduct ½ mark
- Stating a correct answer in degrees instead of radians or vice versa, deduct 1 mark
- Introducing a variable without defining it, deduct ½ mark

Ex.: $2\sin x = 1$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \text{ over } [0, 2\pi]$$

$$x = \frac{\pi}{6} + 2k\pi \text{ or } \frac{5\pi}{6} + 2k\pi$$

Without an explanation of "k", deduct ½ mark. Student can write $k \in \mathrm{I}$ or k is an integer.

- Equating LHS and RHS throughout the proof of an identity, deduct 1 mark
- Omitting two or more θs in a trig identity or equation, deduct ½ mark
- Changing a variable without redefining it, in either an equation or an identity, deduct ½ mark for poor notation

Ex.: $2\sin\theta = 1$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

or

$$\cos^2\theta = 1$$

$$x^2 = 1$$

$$x = \pm 1$$

- Writing $\sin x^2$ instead of $\sin^2 x$, deduct ½ mark

- Incorrect precision or rounding, deduct ½ mark
- In a binomial theorem expansion, giving the answer as an entire term when only the numerical coefficient is requested, deduct ½ mark
- In using the log of a power theorem and omitting parentheses such as:

  a) $\log 3^{x+1} = x+1 \log 3 = x \log 3 + \log 3$ (deduct ½ mark)

  b) $\log 3^{x+1} = x+1 \log 3 = x + \log 3$ (deduct 1 mark)

- Unsimplified fractions such as: $\frac{\frac{1}{2}+\frac{1}{3}}{2}$, deduct ½ mark
- Unreduced fractions such as: $\frac{6}{8}$ or unrationalized fractions such as: $\frac{\sqrt{3}}{\sqrt{2}-1}$ are acceptable.
- If a student simplifies fractions and makes an error, deduct ½ mark.
- Teachers should be aware that in all functions, unless otherwise stated, the domain is the largest subset of the real numbers for which the function is meaningful.
- Permutations or combinations that use cases must have the cases briefly described, otherwise, deduct ½ mark.
- Omitting brackets in a fraction but assuming them, deduct ½ mark

  Ex.: $\frac{2^2}{3} = \frac{4}{9}$ (deduct ½ mark)

Appendix B

IRREGULARITIES IN STANDARDS TESTS

A GUIDE FOR LOCAL MARKING

The *Administration Manual for Supervising Teachers* and the *Policies and Procedures for Standards Tests* are distributed to teachers and administrators to ensure consistency in the administration of standards tests. These documents deal with the issues of test material security and the department's policy regarding adaptations and exemptions.

During the marking of standards tests, markers have occasionally encountered test booklets in which there have been irregularities. The following list provides examples of such irregularities, and is not intended to be exhaustive:

- Completely different penmanship in the same booklet
- Incoherent work with correct answer(s)
- Notes from a teacher indicating how they have assisted students during the test administration
- Student offering that they received teacher assistance for a question
- Student submitting work on unauthorized paper
- Evidence of plagiarism or cheating

If a student's mark on the test is 0% because he/she left the whole test booklet blank or answered with a combination of non-responses and inappropriate responses, please complete an *Irregular Test Paper Report* to confirm that the result is accurate (i.e., the student was not, in fact, absent or exempted).

Please note that student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up action at the school level. In this case, please ensure the department is made aware that a follow-up has taken place by completing an *Irregular Test Paper Report*.

Except in the case of cheating or plagiarism where the result is a standards test mark of 0%, it is the responsibility of the divisions/schools to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Paper Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the Instruction, Curriculum and Assessment Branch along with all other test materials.

Irregular Test Paper Report

Test: _____

Date marked: _____

Booklet ID No.: _____

Problem noted: _____

_____

_____

_____

_____

_____

Question(s) affected: _____

_____

_____

Action taken or rationale for assigning marks: _____

_____

_____

_____

_____

_____

_____

Follow-up: _____

_____

_____

_____

_____

_____

Decision: _____

_____

_____

_____

_____

_____

_____

Marker's Signature: _____

Principal's Signature: _____

For Department Use Only—After Marking Complete

Consultant: _____

Date: _____

Senior 4 Pre-Calculus Mathematics (40S) Standards Test

Student Booklet (Part 1)

June 2005

Manitoba
Building for the Future

Senior 4 Pre-Calculus Mathematics (40S)
Standards Test

DESCRIPTION

Total Possible Marks: 100 **Time: 3 hours**

Part	Description	Time	Value
Part 1	• **Open Response:** 10 questions worth a total of 35 marks Calculators (scientific or graphing) are permitted.	60 minutes	35%
Part 2	• **Selected Response:** 15 questions worth 1 mark each • **Restricted Response:** 15 questions worth 1 mark each • **Open Response:** 12 questions worth a total of 35 marks Calculators are not permitted.	120 minutes	65%

GENERAL DIRECTIONS

- Formulas are given on the tear-out sheet at the beginning of this test booklet. Please do not tear out any pages other than this formula sheet.
- Read all instructions carefully.
- The blank pages at the back of each booklet may be used for rough work. No marks will be given for work done on these pages.
- Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- When the *Student Booklet (Part 2)* is distributed, you may keep the *Student Booklet (Part 1)* in case you wish to continue working on it without a calculator.

Formula Sheet—Pre-Calculus Mathematics (40S)

$$s = \theta r$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\cos 2\alpha = 1 - 2\sin^2\alpha$$

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$P(n,r) \text{ or } {}_nP_r = \frac{n!}{(n-r)!}$$

$$C(n,r) \text{ or } {}_nC_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_nC_k a^{n-k} b^k$$

$$\mathrm{P(A \text{ or } B)} = \mathrm{P(A)} + \mathrm{P(B)} - \mathrm{P(A \text{ and } B)}$$

$$\mathrm{P(A \text{ and } B)} = \mathrm{P(A)} \cdot \mathrm{P(B|A)}$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = Pe^{rt}$$

$$e \approx 2.718\ 28$$

$$\log_a(MN) = \log_a M + \log_a N$$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\log_a(M^n) = n\log_a M$$

$$\log_a M = \frac{\log_b M}{\log_b a}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \quad a > b$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, \quad a > b$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$(y-k) = a(x-h)^2$$

$$(x-h) = a(y-k)^2$$

$$t_n = t_1 r^{n-1}$$

$$S_n = \frac{t_1(1-r^n)}{1-r} = \frac{t_1(r^n-1)}{r-1}$$

$$S_n = \frac{t_1 - t_n r}{1-r}$$

$$S_\infty = \frac{t_1}{1-r}, \quad |r| < 1$$

Open-Response Questions

Instructions

- There are 10 questions for a total of 35 marks.
- Calculators (scientific or graphing) are allowed for this part of the test.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions should include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized and clear.
- If any curve contains asymptotes, the asymptotes should be included in the graph.

DO NOT USE

Open-Response Questions

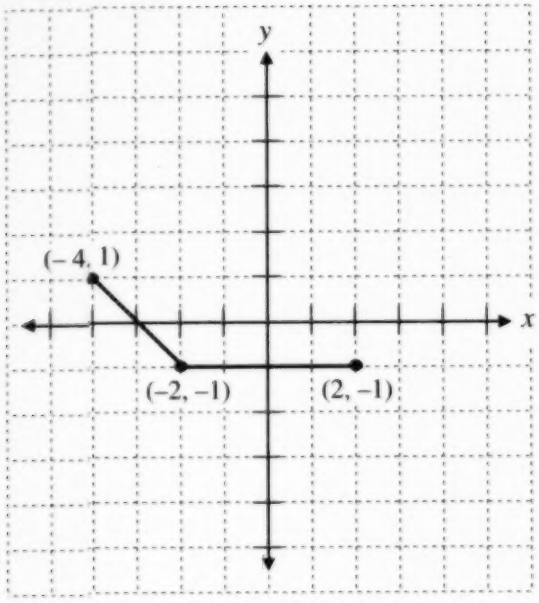
1. Solve the equation $2^{3x+1} = 5^{x+3}$ algebraically. 101

   Express your final answer correct to 3 decimal places.

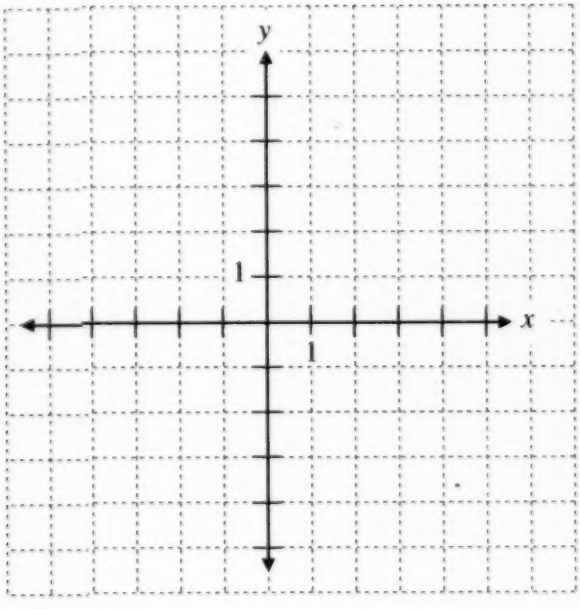
(3 marks)

DO NOT USE

2. Given the graph of $y = f(x)$:



a) Sketch the graph of $y = f(x-1)+2$. (2 marks) 102

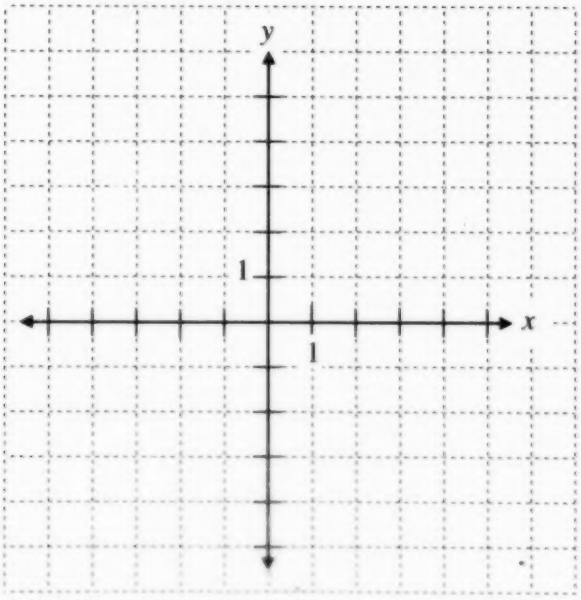


DO NOT USE

b) Sketch the graph of $y = \frac{1}{f(x)}$.

103

(2 marks)



DO NOT USE

104

3. Solve for x algebraically:

$$\log_2(x-2) + \log_2(x+1) - 2 = 0$$

(4 marks)

DO NOT USE

4. Solve for x where $0 \leq x \leq 2\pi$. 105

State the solution(s) correct to 3 decimal places.

$$\sin^2 x + \cos^2 x + \tan^2 x = 3$$

(4 marks)

5. The school indoor soccer team is to consist of 6 girls and 5 boys, chosen at random from a group of 10 girls and 11 boys. 106

Two of the girls and one of the boys are very good friends.

What is the probability that all 3 friends will be on the team?

Briefly explain your calculations.

(3 marks)

DO NOT USE

6. Find the middle term of the expansion of $(3a^2 - 2b)^8$. 107

Simplify your answer.

(3 marks)

7. How many four-digit positive integers greater than 4 700 can be formed using the digits 1, 4, 5, 8, 9 if repetition is not allowed? 108

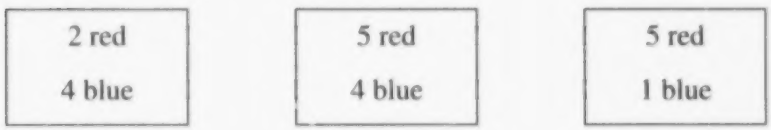
Briefly explain your calculations.

(3 marks)

DO NOT USE

109

8. Three boxes contain red marbles and blue marbles as shown below.



Box 1 Box 2 Box 3

Jack throws a standard six-sided die to choose a box.

If the die shows 1 or 2 he chooses Box 1.

If the die shows 3, 4 or 5 he chooses Box 2.

If the die shows 6 he chooses Box 3.

He then chooses a marble at random from the box which he selected.

Find the probability that the marble is red.

(4 marks)

DO NOT USE

110

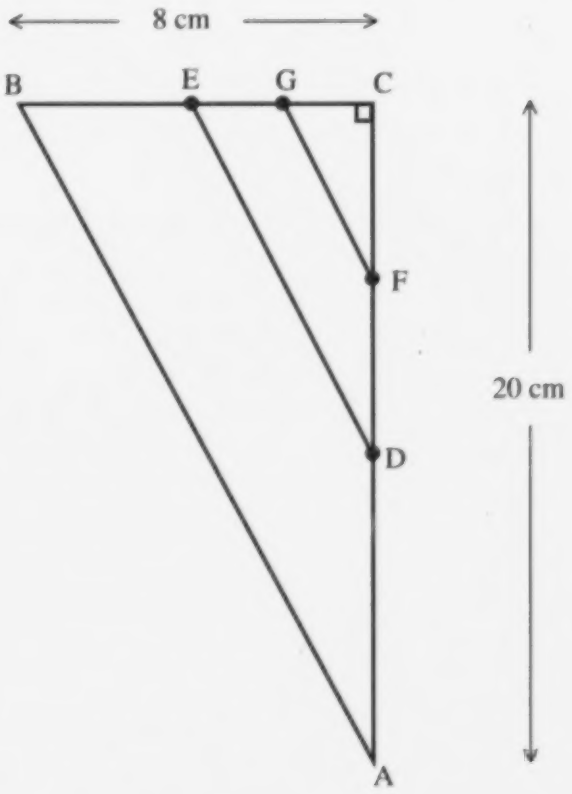
9. ΔABC is a right triangle with $\overline{BC}$ having a length of 8 cm, and $\overline{AC}$ having a length of 20 cm.

New right triangles are formed by joining the midpoints of the perpendicular sides and retaining the right angles as shown in the diagram ($\Delta ABC, \Delta DEC, \Delta FGC$ and so on).

This process continues an infinite number of times.

Find the sum of the areas of these triangles in cm^2.

(3 marks)



DO NOT USE

10. a) Prove the following identity:

$$\frac{1+\cos 2x}{\sin 2x}=\cot x$$

(3 marks)

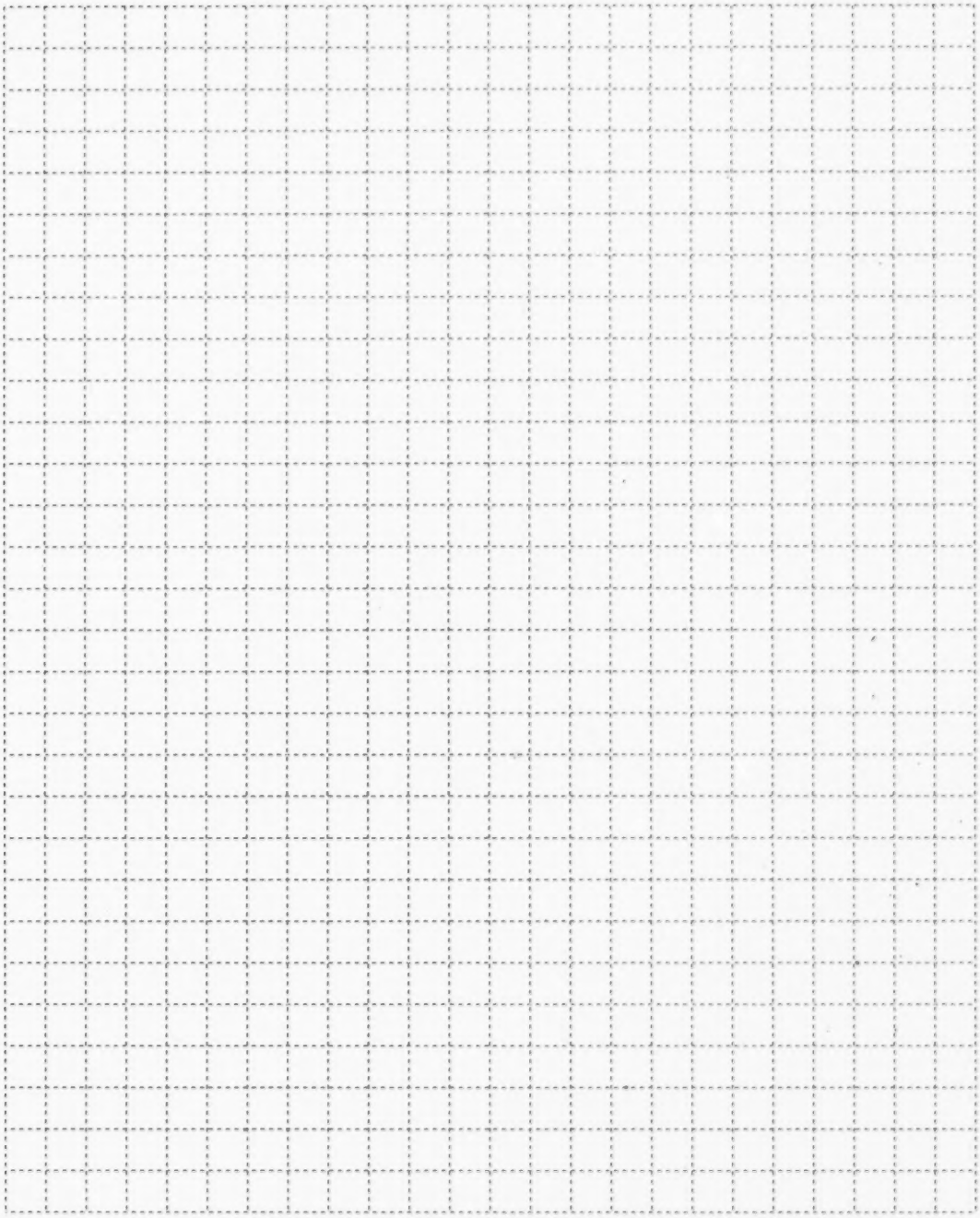
111

b) Give one value of x where $\frac{1+\cos 2x}{\sin 2x}$ is undefined.

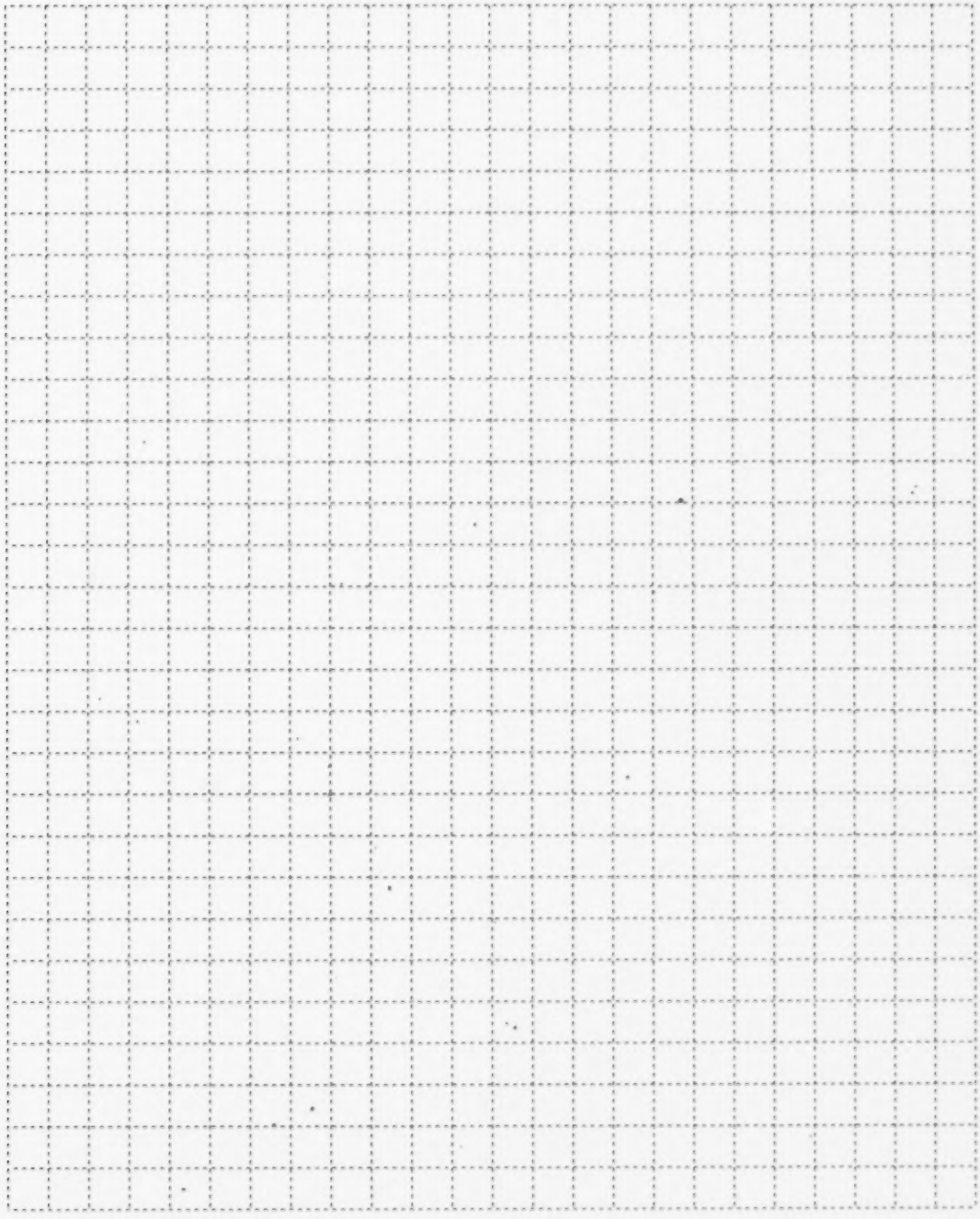
(1 mark)

112

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.



NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.



NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

Copyright © 2005, the Crown in Right of Manitoba,
as represented by the Minister of Education, Citizenship and Youth.
Manitoba Education, Citizenship and Youth,
Instruction, Curriculum and Assessment Branch,
Winnipeg, Manitoba R3E 3J5

Permission is hereby given by the copyright owner to reproduce this
document on a non-profit basis for educational purposes provided
the source is mentioned.

Senior 4 Pre-Calculus Mathematics (40S) Standards Test

Student Booklet (Part 2)

June 2005

Manitoba

Building for the Future

Copyright © 2005, the Crown in Right of Manitoba,
as represented by the Minister of Education, Citizenship and Youth.
Manitoba Education, Citizenship and Youth,
Instruction, Curriculum and Assessment Branch,
Winnipeg, Manitoba R3E 3J5

Permission is hereby given by the copyright owner to reproduce this
document on a non-profit basis for educational purposes provided
the source is mentioned.

Selected-Response Questions

Instructions

- There are 15 selected-response questions.
- Each question is worth 1 mark.
- Read each question carefully.
- Examine all the choices before you decide on your answer.
- You may use the spaces beside each question for rough work.
- Record your answers on the sheet provided.
- Provide only one answer per question.
- There is no penalty for guessing.
- Answer all questions.
- Calculators (scientific or graphing) are **not** allowed for this part of the test.

Selected-Response Questions

11. If $\sin(A) = \frac{-5}{13}$ and $\cos(A) = \frac{12}{13}$, then the point $P(A)$, on the unit circle, is located in quadrant:

    A) I

    B) II

    C) III

    D) IV

12. If $\sin\theta = -1$, which of the following equations is true?

    A) $\cot\theta = 1$

    B) $\csc\theta = 1$

    C) $\tan\theta = 0$

    D) $\cos\theta = 0$

13. The number of solutions for the equation $\sin x = -\frac{1}{2}$ in the interval $[0, 8\pi]$ is:

    A) 2

    B) 4

    C) 8

    D) 16

14. The period of the function f is 10.
If $f(0)=16$, $f(5)=8$, and $f(10)=16$, then the value of $f(15)$ is:

A) 0

B) 8

C) 16

D) 24

15. If a wheel of radius 2 m rolls 10 m, it has turned through an angle of:

A) $5°$

B) $20°$

C) $\left(\frac{5\pi}{180}\right)^{\circ}$

D) $\left(\frac{900}{\pi}\right)^{\circ}$

16. If the graph of $y=f(x)$ is stretched vertically by a factor of 2 and moved down 3 units, an equation for the transformed graph would be:

A) $y=2f(x)-3$

B) $y=2f(x)+3$

C) $y=f(2x)-3$

D) $y=f(2x)+3$

17. Consider the graph of $y = x^2$.

The effect on this graph of replacing x with $(x + 7)$ will be to shift the graph 7 units:

A) right

B) left

C) up

D) down

18. For positive values m, n, and p, the expression $\log m + \frac{1}{3}(\log n - \log p)$ is equivalent to:

A) $\log\left(m\sqrt[3]{\frac{n}{p}}\right)$

B) $\log\left(\frac{mn}{3p}\right)$

C) $\log\left(m + \frac{1}{3}(n - p)\right)$

D) $\log\left(m\sqrt[3]{n - p}\right)$

19. Evaluate: $\sum_{k=3}^{5} 2k^2$

A) 68

B) 100

C) 110

D) 200

20. A bag contains 4 black, 6 green and 2 yellow marbles.

If one marble is randomly drawn, the probability that it is **not** black is:

A) $\frac{1}{8}$

B) $\frac{1}{6}$

C) $\frac{1}{2}$

D) $\frac{2}{3}$

21. Twelve students will ride in two vehicles. In how many ways can a teacher choose 7 students to ride in the van and 5 students to ride in the car?

A) $7!5!$

B) ${}_{12}P_7 \times {}_5P_5$

C) ${}_{12}C_7 \times {}_{12}C_5$

D) ${}_{12}C_7 \times {}_5C_5$

22. An expression equivalent to $\frac{(n+3)!}{(n+1)(n!)}$ is:

A) $\frac{(n+3)(n+2)}{n}$

B) $\frac{(n+3)(n+2)}{n!}$

C) $(n+3)(n+2)$

D) $(n+3)(n+2)(n+1)$

23. Identify the conic section defined by the following equation:

$$x^2 + 4y^2 - 12x + 2y - 8 = 0$$

A) parabola

B) ellipse

C) hyperbola

D) circle

24. An equation for a circle tangent to the x-axis at $(3, 0)$ and tangent to the y-axis at $(0, -3)$ is:

A) $(x-3)^2 + (y+3)^2 = 9$

B) $(x-3)^2 + (y+3)^2 = 3$

C) $(x+3)^2 + (y-3)^2 = 9$

D) $(x+3)^2 + (y-3)^2 = 3$

25. The graph of the conic section represented by the equation $(y-3)^2 = -3(x+2)$ opens:

A) up

B) down

C) left

D) right

Restricted-Response Questions

Instructions

- There are 15 restricted-response questions.
- Each question is worth 1 mark.
- Calculators are **not** allowed for this part of the test.
- Write your final answer in the space provided.
- Work need not be shown in order to obtain full marks for a correct answer. However, in the case of an incorrect final answer, part marks may be awarded only if work is shown.
- Solutions need not have rationalized denominators.

DO NOT USE

Restricted-Response Questions

26. A 40° angle is equivalent to an angle of _____ radians. 113

Write your answer as an exact value.

(1 mark)

27. The exact value of $\sin^{-1}\left[\sin\left(\frac{5\pi}{6}\right)\right]$ is: 114

(1 mark)

28. Find the exact value of $2\cos^2\left(\frac{\pi}{8}\right)-1$. 115

(1 mark)

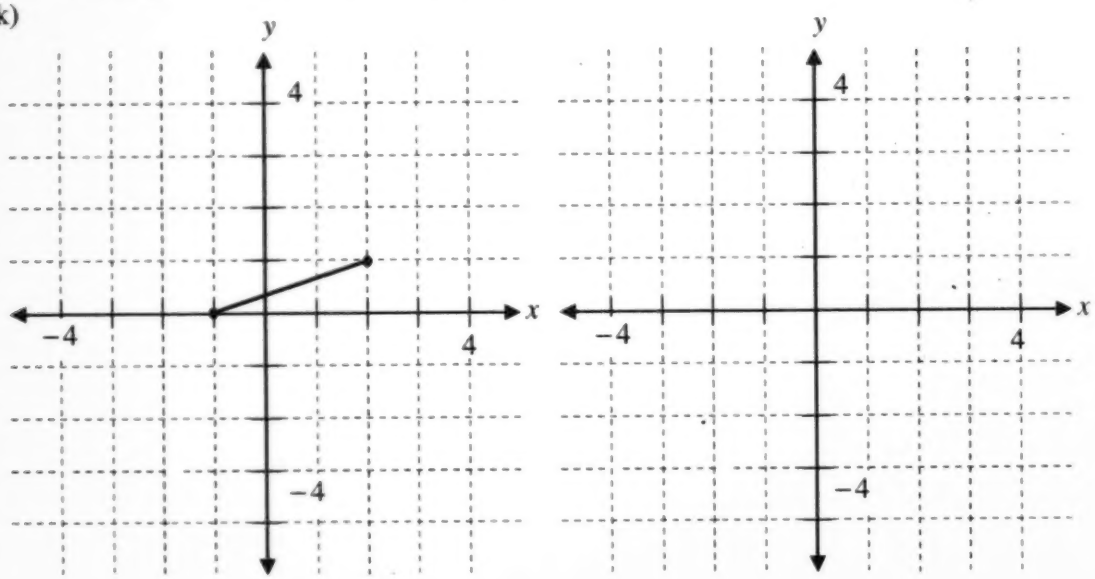
DO NOT USE

29. Write an equation for one vertical asymptote of the graph of $y = \sec x$. (1 mark) 116

30. The graph of the function $y = g(x)$ is shown on the left. 117

On the grid at right, sketch a clearly labelled graph of $y = -g(x)$.

(1 mark)



DO NOT USE

31. Solve for x: 118

$$2^{3x-1} = 4^{x+1}$$

(1 mark)

32. Given $f(x) = \log(x+10)+3$, evaluate $f(0)$. 119

(1 mark)

33. In how many ways can all the letters of the word T A B L E be arranged if the vowels and consonants must alternate? 120

(1 mark)

DO NOT USE

34. Simplify: 121

$$\frac{{}_{500}C_{400}}{{}_{500}C_{100}}$$

(1 mark)

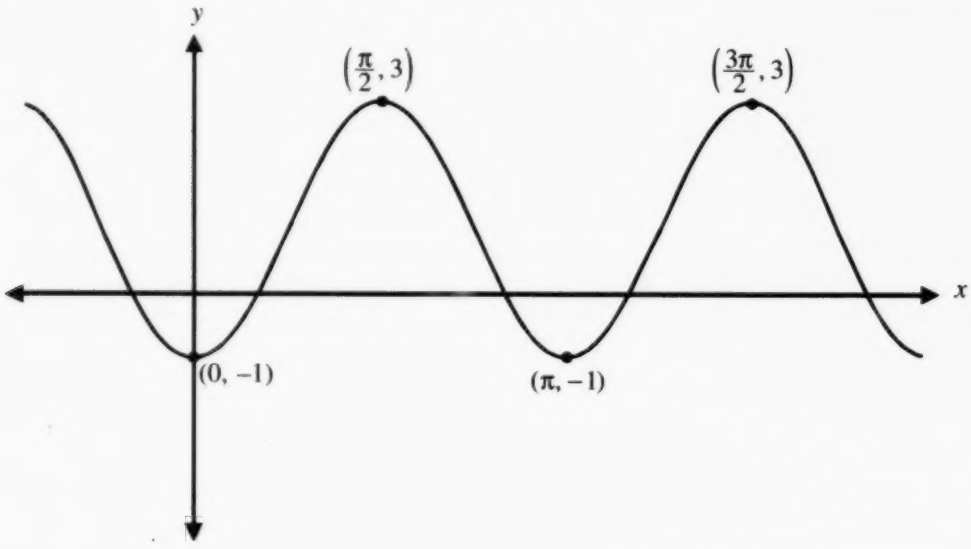
35. An ellipse, centred at the origin, has x-intercepts of 3 and –3 and y-intercepts of 4 and –4. 122
Write its equation.

(1 mark)

DO NOT USE

36. What is the amplitude of the function sketched below? 123

(1 mark)



amplitude _____

37. Find the value of the expression: 124

$$\log_3\left(\log_5(125)\right)$$

(1 mark)

DO NOT USE

38. The point (27, 3) lies on the graph of $y = \log_b x$. 125

The exact value of b is:

(1 mark)

39. Using **all** the letters in the word E X C E L L E N T, indicate the number of possible distinguishable arrangements that can be made. 126
Leave your answer in factorial form.

(1 mark)

40. State the position of the term containing x^{30} in the expansion of $\left(2x^3 + 4\right)^{12}$. 127

(1 mark)

Open-Response Questions

Instructions

- There are 12 questions worth a total of 35 marks.
- Calculators are **not** allowed for this part of the test.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Your solutions should be neat, clear and well organized.
- If any curve contains asymptotes, the asymptotes should be included in the graph.
- Solutions need not have rationalized denominators.

DO NOT USE

Open-Response Questions

41. P is the point of intersection of the unit circle and the line segment joining the origin and the point $(3, -2)$. Find the coordinates for P. 128

(3 marks)

42. The graph of the conic section whose equation is $\frac{x^2}{4} + \frac{y^2}{1} = 1$ is translated 3 units to the right.

a) Write an equation of the new conic section.

(1 mark) 129

b) State the domain of the new conic section.

(1 mark) 130

c) State the range of the new conic section.

(1 mark) 131

DO NOT USE

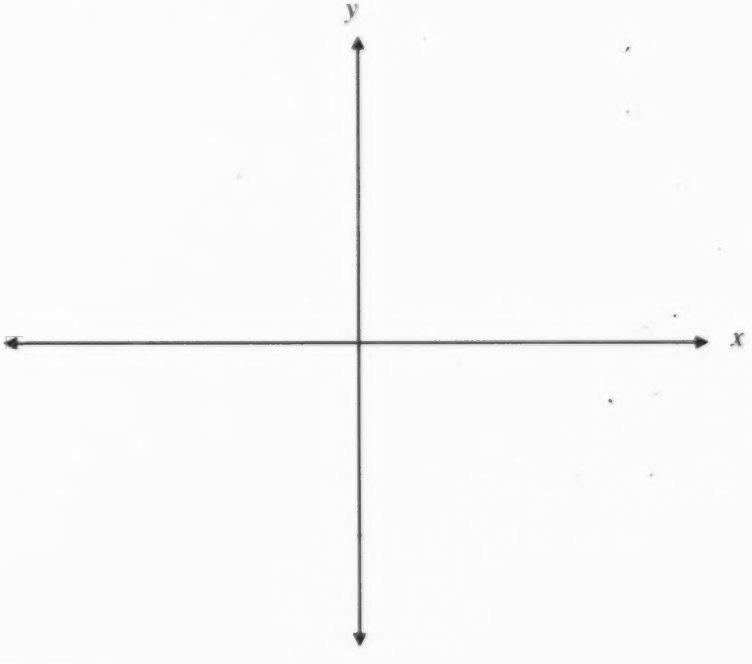
43. If $\sec\theta = -3$ and $\tan\theta > 0$, find the value of $\sin\theta$. 132

(3 marks)

44. Sketch a clearly labelled graph of the following function. 133
State the y-intercept and the equation(s) of any asymptotes.

$y = e^x + 3$

(3 marks)



y-intercept _____

asymptote(s) _____

DO NOT USE

45. Given $f(x) = \log_2(x-3)$, find an equation for $f^{-1}(x)$. 134

(3 marks)

46. The probability of the chemistry teacher giving a test next Tuesday is 0.2 and the probability of the mathematics teacher giving a test next Tuesday is 0.4.

a) Find the probability that next Tuesday there will be a test on chemistry and a test on mathematics. 135

(1 mark)

b) Find the probability that next Tuesday there will be a test on chemistry or a test on mathematics. 136

(2 marks)

DO NOT USE

47. Determine the coordinates of the vertices of the conic section with the following equation: 137

$$9x^2 - 4y^2 - 8y - 40 = 0$$

(3 marks)

48. Express in simplest form: 138

$$\frac{\left(\cos x + \dfrac{\sin^2 x}{\cos x}\right)}{\sec x}$$

(2 marks)

DO NOT USE

49. Find the exact value of:

a) $\sin\left(\frac{5\pi}{12}\right)$ 139

(3 marks)

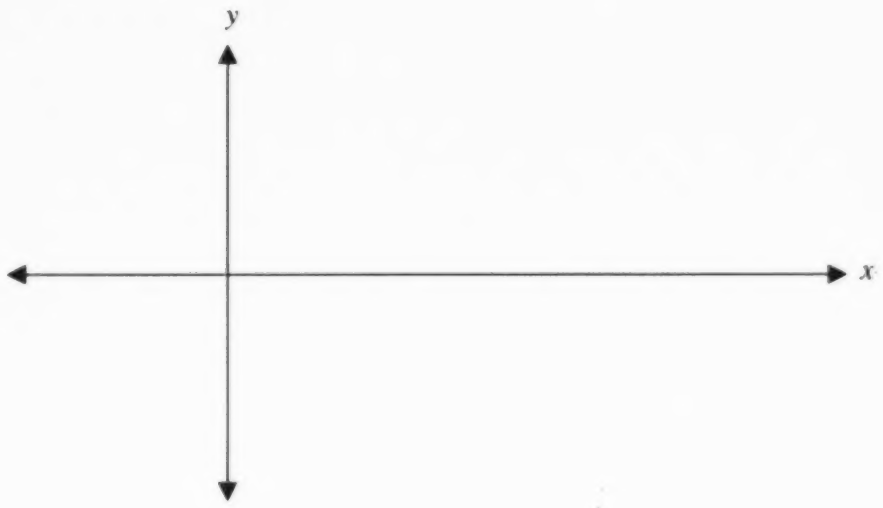
b) $\csc\left(\frac{5\pi}{12}\right)$ 140

(1 mark)

DO NOT USE

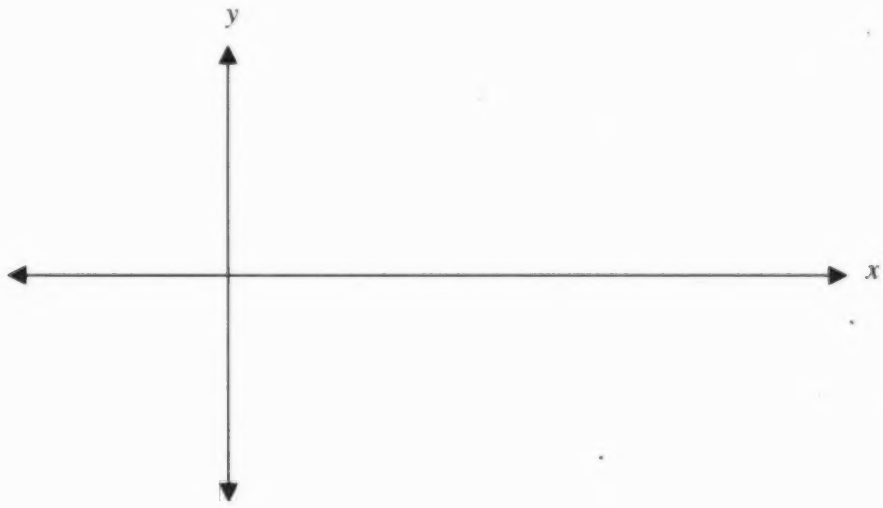
50. a) Sketch a clearly labelled graph of at least one period of $f(x)=3\cos\left(\frac{\pi}{2}(x+1)\right)$. 141

(3 marks)



b) Sketch $y=|f(x)|$. 142

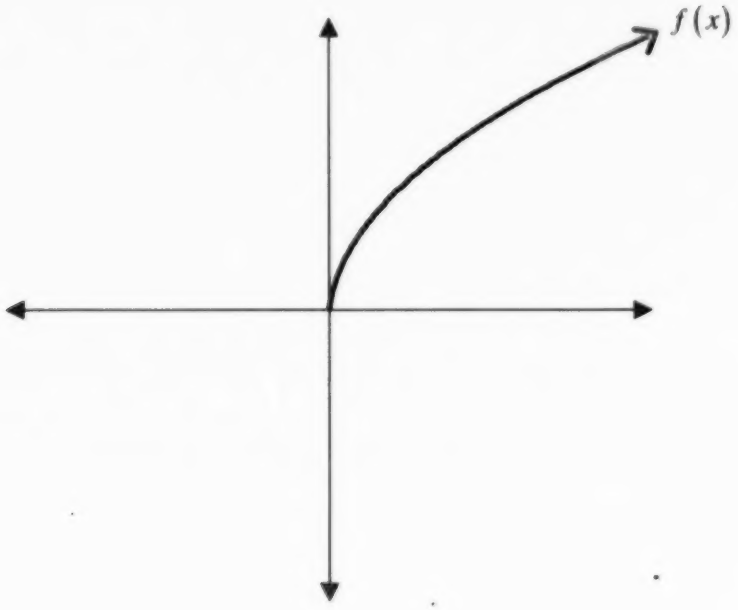
(1 mark)



DO NOT USE

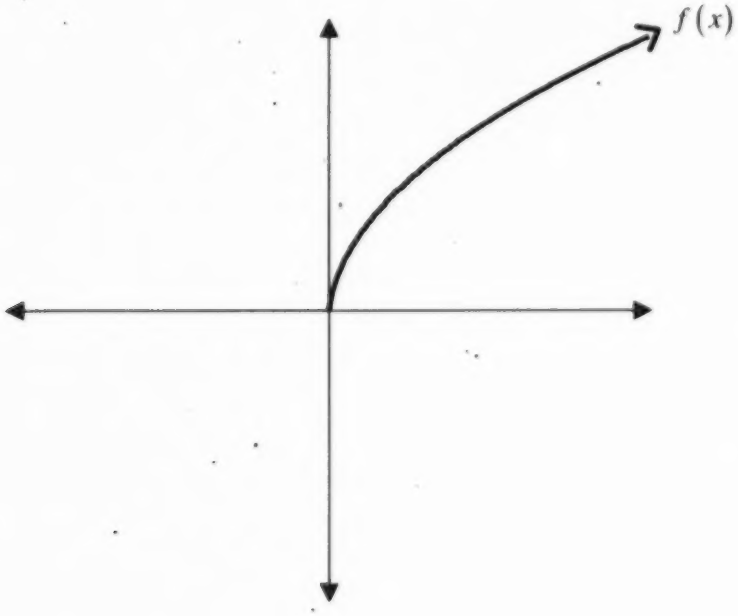
51. a) Complete the sketch of $f(x)$ below if $f(x)$ is even. 143

(1 mark)



b) Complete the sketch of $f(x)$ below if $f(x)$ is odd. 144

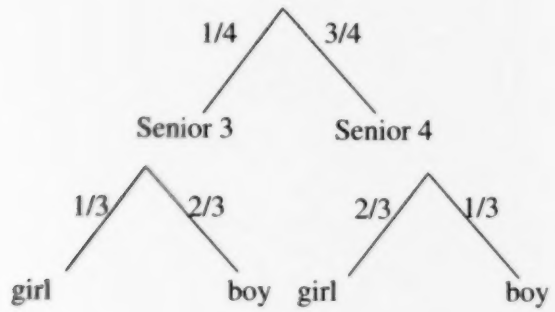
(1 mark)



DO NOT USE

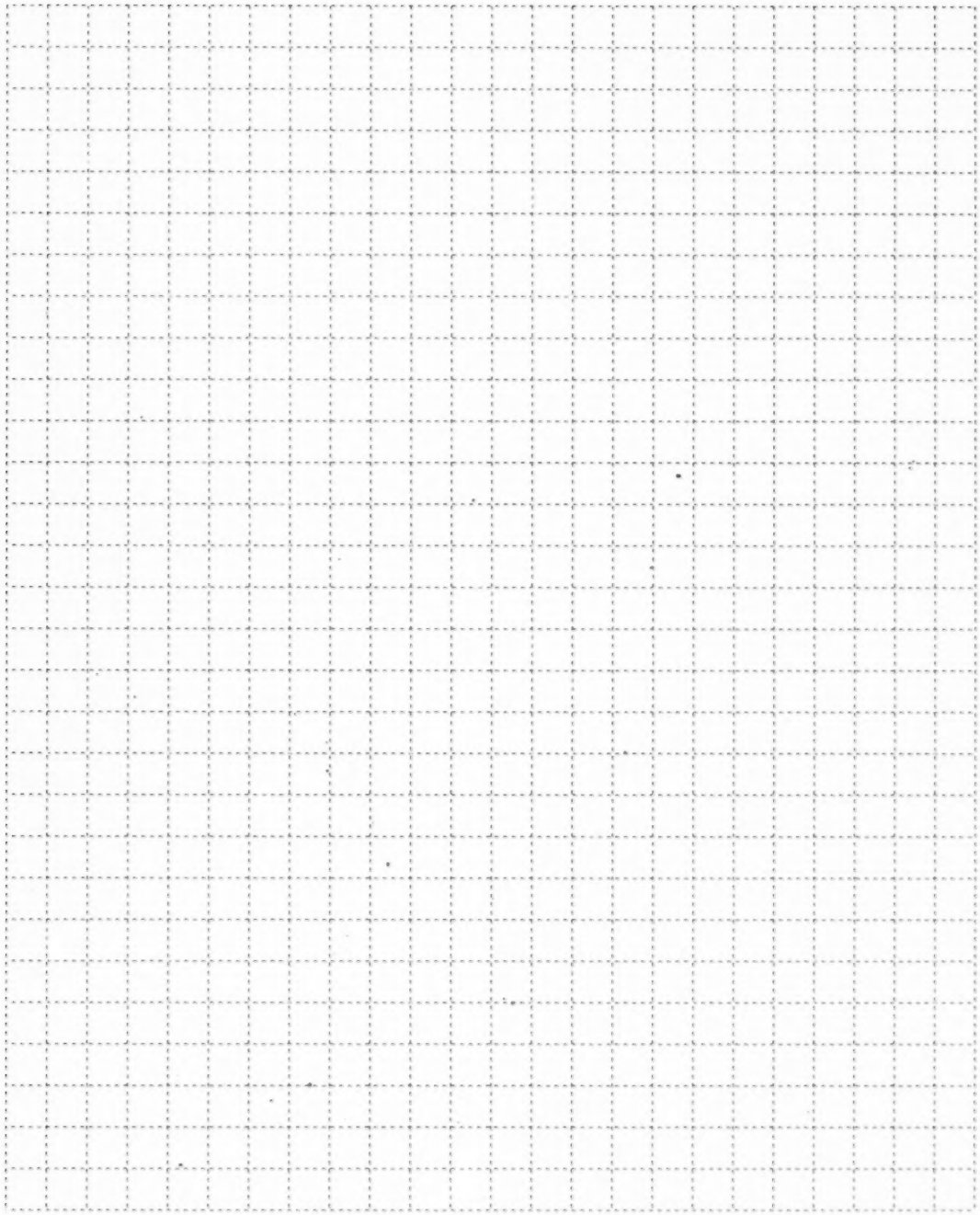
145

52. From a group of Senior 3 and Senior 4 students, a principal is going to choose one student at random. The tree diagram below illustrates the probabilities. For example, the probability of selecting a Senior 3 student is $\frac{1}{4}$.

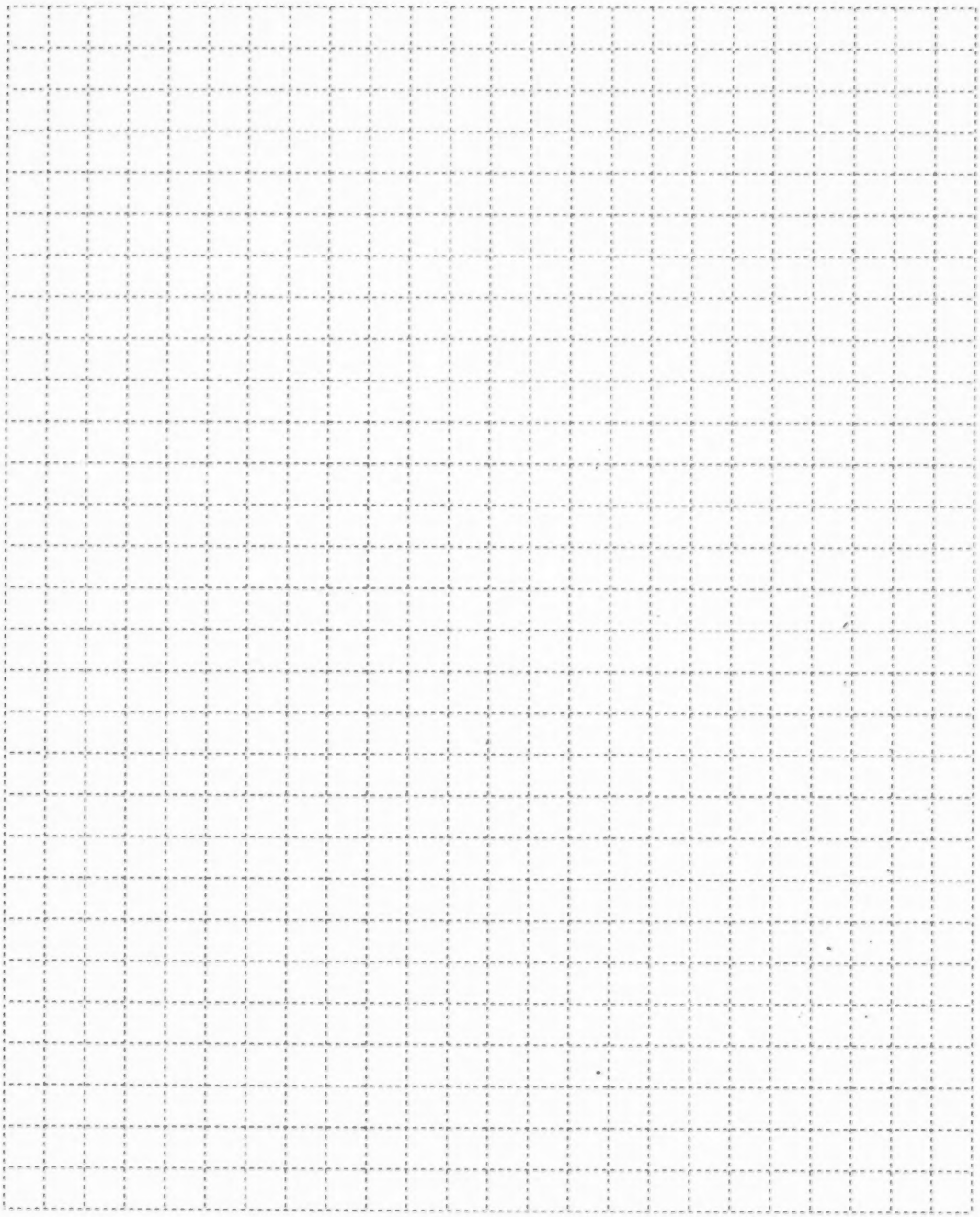


If the principal chooses a girl, what is the probability that the girl is in Senior 3?
(2 marks)

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.



NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.



NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE.